From invariant manifolds to local embedding techniques: a path to finite-dimensional reduction of PDE's

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Dimension reduction of partial differential equations poses intriguing challenges when framed in a dynamical system setting. Finite-dimensional dissipative systems typically exhibit lower dimensional invariant geometrical structures containing their stable/slow features and which can be exploited for dimension reduction. Their infinite dimensional counterparts, such as reaction diffusion equations, can themselves exhibit similar features under the form of countably many invariant manifolds.

In this communication we address the influence of diffusion in modifying the structure of such manifolds for prototypical reaction-diffusion systems, by considering model systems for which analytical solutions can be obtained. Furthermore, we extend the analysis of manifold bifurcations developed in [1], and based on compactification (Poincaré projection) to combustion models in the presence of diffusion and of a first order exothermic reaction.

The reconstruction of intrinsic time-scale spectra (Lyapunov spectra) and the ensuing determination of invariant subspaces is often a greater challenge for infinite-dimensional systems than the direct numerical integration itself. In the light of model reduction, the requirement of invariance can be relaxed and attempts can be made to locally and approximately embed the dynamics (and the related invariant subspaces) within non-invariant linear subspaces of variable dimension, built on the basis of local dynamical features (e.g. stretching porperties, viewed in the framework of an approximate and local definition of normal hyperbolicity). System dynamics can be constrained to such locally linear embeddings, hence inheriting their lower-dimensional features. This is the essence of the Stretching-Based-Reduction (SBR) method proposed in [2], which shares computational simplicity, with a simplified geometric description grounded on a local characterization of normal hyperbolicity.

The aim of this presentation is to provide an overview of the connections and differences between the estimate of invariant geometric features in reaction-diffusion systems and the formulation of efficient reduction strategies for infinite-dimensional dynamical systems.

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