New Conjectures about Zeroes of Riemann's Zeta Function

Yuri Matiyasevich

St.Petersburg Department
of Steklov Institute of Mathematics
of Russian Academy of Sciences

http://logic.pdmi.ras.ru/~yumat

Blowing the Trumpet

"The physicist George Darwin used to say that every once in a while one should do a completely crazy experiment, like blowing the trumpet to the tulips every morning for a month. Probably nothing would happen, but what if it did?"

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The series converges for $s > 1$ and diverges at $s = 1$:

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Other values of $\zeta(s)$ found by EULER

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\zeta(2) &= \frac{1}{6}\pi^2 \\
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\zeta(6) &= \frac{1}{945}\pi^6 \\
\zeta(8) &= \frac{1}{9450}\pi^8 \\
\zeta(10) &= \frac{691}{638512875}\pi^{10} \\
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Bernoulli numbers

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$$\frac{x}{e^x - 1} = \sum_{k=0}^{\infty} \frac{1}{k!} \mathrm{B}_k x^k$$

$$\mathrm{B}_0 = 1, \quad \mathrm{B}_2 = \frac{1}{6}, \quad \mathrm{B}_4 = -\frac{1}{30}, \quad \mathrm{B}_6 = \frac{1}{42}, \quad \mathrm{B}_8 = -\frac{1}{30},$$

$$\mathrm{B}_{10} = \frac{5}{66}, \quad \mathrm{B}_{12} = -\frac{691}{2730}, \quad \mathrm{B}_{14} = \frac{7}{6}, \quad \mathrm{B}_{16} = -\frac{3617}{510}$$

$$\mathrm{B}_1 = -\frac{1}{2}, \quad \mathrm{B}_3 = \mathrm{B}_5 = \mathrm{B}_7 = \mathrm{B}_9 = \mathrm{B}_{11} = \cdots = 0$$

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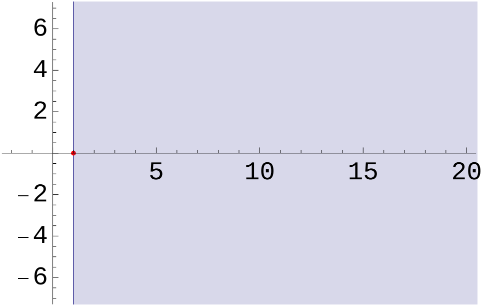

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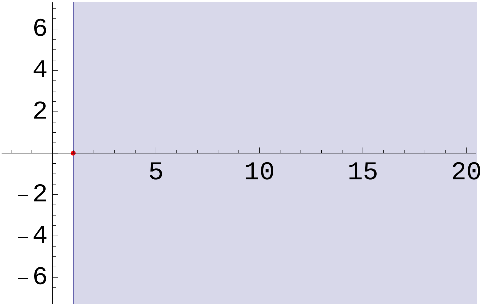

The series converges in the semiplane $\Re(s) > 1$ and defines a function that can be analytically extended to the entire complex plane except for the point $s = 1$, its only (and simple) pole.

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Riemann:

$$\zeta(1-s) \quad = \quad \cos\left(\frac{\pi s}{2}\right) 2^{1-s} \pi^{-s} \Gamma(s) \zeta(s) \qquad s = \sigma + \mathrm{i}t$$

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Function $\xi(s)$ is entire, its zeroes are exactly *non-trivial* (i.e., non-real) zeroes of $\zeta(s)$.

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$$\begin{aligned}\psi(x) &= \sum_{\substack{q \leq x \\ q \text{ is a power} \\ \text{of prime } p}} \ln(p) \\ &= \ln(\mathrm{LCM}(1, 2, ..., \lfloor x \rfloor))\end{aligned}$$

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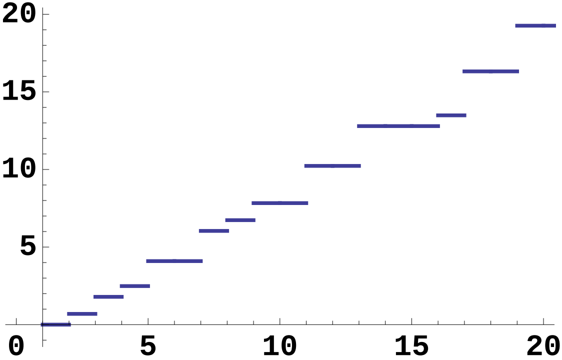

Von Mangoldt Theorem

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Theorem (Hans Carl Fridrich von Mangoldt [1895]). *For any non-integer $x > 1$*

$$\psi(x) = x - \sum_{\xi(\rho)=0} \frac{x^\rho}{\rho} - \sum_n \frac{x^{-2n}}{-2n} - \ln(2\pi)$$

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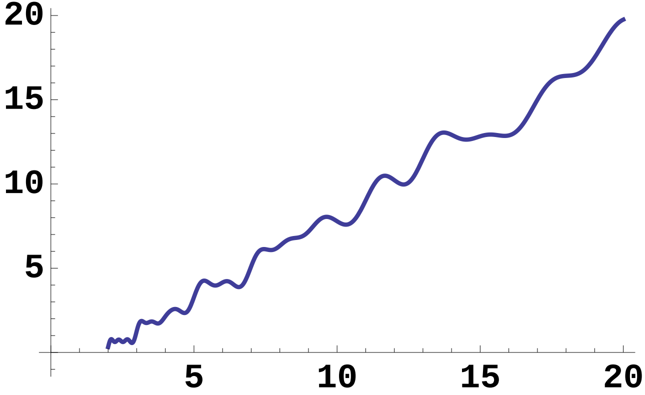
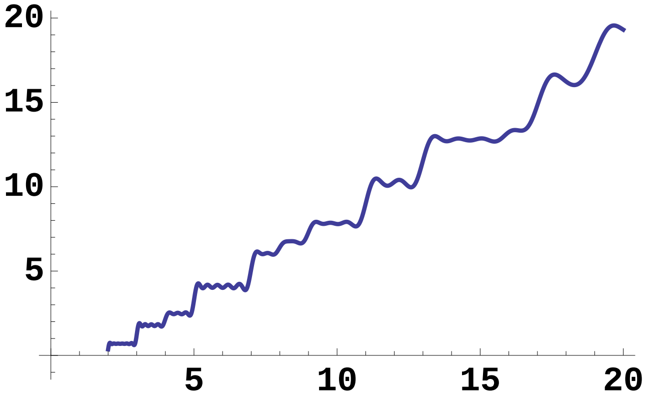
$$x - \sum_{\substack{\xi(\rho)=0 \\ |\rho|<50}} \frac{x^{\rho}}{\rho} - \sum_{n} \frac{x^{-2n}}{-2n} - \ln(2\pi)$$



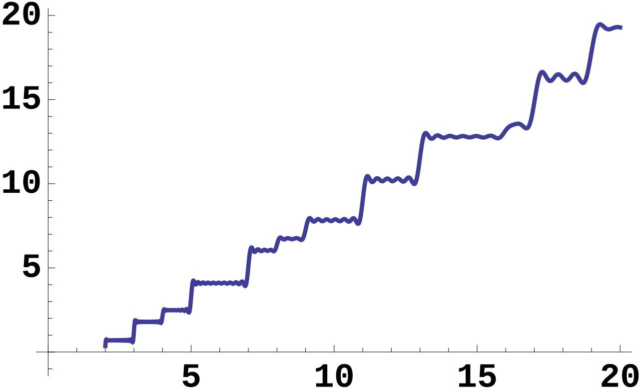
Von Mangoldt Theorem

$$x - \sum_{\substack{\xi(\rho)=0 \\ |\rho|<100}} \frac{x^\rho}{\rho} - \sum_n \frac{x^{-2n}}{-2n} - \ln(2\pi)$$



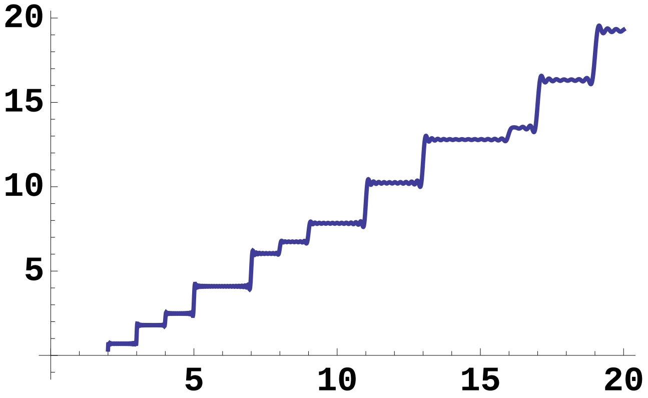
Von Mangoldt Theorem

$$x - \sum_{\substack{\xi(\rho)=0 \\ |\rho|<200}} \frac{x^\rho}{\rho} - \sum_n \frac{x^{-2n}}{-2n} - \ln(2\pi)$$



Von Mangoldt Theorem

$$x - \sum_{\substack{\xi(\rho)=0 \\ |\rho|<400}} \frac{x^{\rho}}{\rho} - \sum_{n} \frac{x^{-2n}}{-2n} - \ln(2\pi)$$



Blowing the Trumpet

Assuming that all zeroes of $\Xi(t)$ are real and simple, let them be denoted $\pm\gamma_1, \pm\gamma_2, \ldots$ with $0 < \gamma_1 < \gamma_2 < \ldots$, thus the non-trivial zeroes of $\zeta(s)$ are $\frac{1}{2} \pm \mathrm{i}\gamma_1, \frac{1}{2} \pm \mathrm{i}\gamma_2, \ldots$

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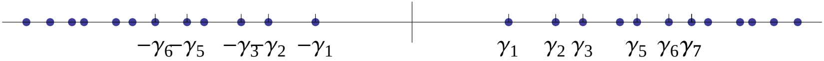

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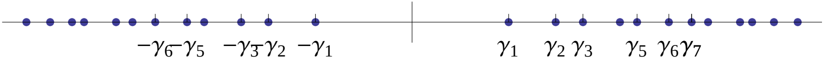

$$\gamma_1 = 14.1347\ldots \qquad \gamma_2 = 21.0220\ldots$$
$$\gamma_3 = 25.0109\ldots \qquad \gamma_4 = 30.4249\ldots$$

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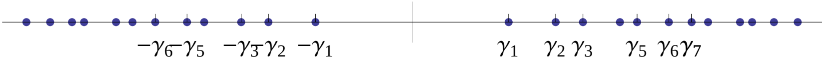

$$\gamma_1 = 14.1347\dots \qquad \gamma_2 = 21.0220\dots$$
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Suppose that we have found $\gamma_1, \gamma_2, \dots, \gamma_{N-1}$; *how could these numbers be used for calculating an (approximate) value of the next zero γ_N?*

Interpolating determinant

Let us try to approximate $\Xi(t)$ by some simpler function also having zeroes at the points $\pm\gamma_1, \ldots, \pm\gamma_{N-1}$.
Consider an interpolating determinant with even functions $f_1, f_2, \ldots$

$$\begin{vmatrix} f_1(\gamma_1) & \ldots & f_1(\gamma_{N-1}) & f_1(t) \\ \vdots & \ddots & \vdots & \vdots \\ f_N(\gamma_1) & \ldots & f_N(\gamma_{N-1}) & f_N(t) \end{vmatrix}$$

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Selecting $f_n(t) = t^{2(n-1)}$ we would obtain just an interpolating polynomial

$$C \prod_{n=1}^{N-1} (t^2 - {\gamma_n}^2)$$

having no other zeros.

Interpolating determinant

Interpolating determinant

If $\gamma_1^*, \gamma_2^*, \ldots$ are zeros of the function

$$\Xi^*(t) = \sum_{k=1}^{N} f_k(t)$$

then the determinant

$$\begin{vmatrix} f_1(\gamma_1^*) & \ldots & f_1(\gamma_{N-1}^*) & f_1(t) \\ \vdots & \ddots & \vdots & \vdots \\ f_N(\gamma_1^*) & \ldots & f_N(\gamma_{N-1}^*) & f_N(t) \end{vmatrix}$$

vanish at every zero of $\Xi^*(t)$.

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$$\Xi(t) = \sum_{n=1}^{\infty} \alpha_n(t) \qquad \alpha_n(t) = -\frac{\pi^{-\frac{1}{4}-\frac{\mathrm{i}t}{2}} \left(t^2 + \frac{1}{4}\right) \Gamma\left(\frac{1}{4} + \frac{\mathrm{i}t}{2}\right)}{2n^{\frac{1}{2}+\mathrm{i}t}}$$

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for $\Im(t) < -\frac{1}{2}$

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Our interpolating determinant

$$\Delta_N(t) = \begin{vmatrix} \beta_1(\gamma_1) & \dots & \beta_1(\gamma_{N-1}) & \beta_1(t) \\ \vdots & \ddots & \vdots & \vdots \\ \beta_N(\gamma_1) & \dots & \beta_N(\gamma_{N-1}) & \beta_N(t) \end{vmatrix}$$

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Blowing the trumpet: the outcome

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$$\Delta_{40}(\mathbf{122.946829}) = -1.86119\ldots \cdot 10^{-926} < 0$$
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$$\Delta_{220}(\mathbf{427.208825084074}) = -1.92776\ldots\cdot 10^{-17793} < 0$$

$$\gamma_{220} = \mathbf{427.208825084074}58052814\ldots$$

$$\Delta_{220}(\mathbf{427.208825084075}) = +9.85564\ldots\cdot 10^{-17794} > 0$$

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$$\gamma_{400} = \mathbf{679.742197882528217719525938911269995 34}56135514\ldots$$

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$$\Delta_{220}(\mathbf{427.208825084075}) = +9.85564\ldots\cdot 10^{-17794} > 0$$

$$\Delta_{400}(\mathbf{679.742197882528217719525938911269995 34}) = -2.95319\ldots\cdot 10^{-52001} < 0$$

$$\gamma_{400} = \mathbf{679.74219788252821771952593891126999534}56135514\ldots$$

$$\Delta_{400}(\mathbf{679.74219788252821771952593891126999535}) = +1.78976\ldots\cdot 10^{-52001} > 0$$

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$$\Delta_{40}(\mathbf{124.25681}) = +9.32826\ldots \cdot 10^{-927} > 0$$

$$\gamma_{41} = \mathbf{124.25681}85543457\ldots$$

$$\Delta_{40}(\mathbf{124.25682}) = -2.20319\ldots \cdot 10^{-925} < 0$$

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$$\Delta_{40}(\mathbf{122.946829}) = -1.86119\ldots\cdot 10^{-926} < 0$$
$$\gamma_{40} = \mathbf{122.946829}29355258\ldots$$
$$\Delta_{40}(\mathbf{122.946830}) = +2.39694\ldots\cdot 10^{-926} > 0$$

$$\Delta_{40}(\mathbf{124.25681}) = +9.32826\ldots\cdot 10^{-927} > 0$$
$$\gamma_{41} = \mathbf{124.25681}85543457\ldots$$
$$\Delta_{40}(\mathbf{124.25682}) = -2.20319\ldots\cdot 10^{-925} < 0$$

$$\Delta_{40}(\mathbf{127.516}) = -2.24237\ldots\cdot 10^{-924} < 0$$
$$\gamma_{42} = \mathbf{127.516}68387959\ldots$$
$$\Delta_{40}(\mathbf{127.517}) = +8.95138\ldots\cdot 10^{-925} > 0$$

Blowing the trumpet: the outcome

$$\Delta_{40}(\mathbf{122.946829}) = -1.86119\ldots \cdot 10^{-926} < 0$$
$$\gamma_{40} = \mathbf{122.946829}29355258\ldots$$
$$\Delta_{40}(\mathbf{122.946830}) = +2.39694\ldots \cdot 10^{-926} > 0$$

$$\Delta_{40}(\mathbf{124.25681}) = +9.32826\ldots \cdot 10^{-927} > 0$$
$$\gamma_{41} = \mathbf{124.25681}85543457\ldots$$
$$\Delta_{40}(\mathbf{124.25682}) = -2.20319\ldots \cdot 10^{-925} < 0$$

$$\Delta_{40}(\mathbf{127.516}) = -2.24237\ldots \cdot 10^{-924} < 0$$
$$\gamma_{42} = \mathbf{127.516}68387959\ldots$$
$$\Delta_{40}(\mathbf{127.517}) = +8.95138\ldots \cdot 10^{-925} > 0$$

$$\Delta_{40}(\mathbf{129.5787}) = +2.78973\ldots \cdot 10^{-926} > 0$$
$$\gamma_{43} = \mathbf{129.5787}04199956\ldots$$
$$\Delta_{40}(\mathbf{129.5788}) = -9.94403\ldots \cdot 10^{-927} < 0$$

Blowing the trumpet: the outcome

$$\Delta_{220}(\mathbf{427.208825084074}) = -1.92776\ldots\cdot 10^{-17793} < 0$$

$$\gamma_{220} = \mathbf{427.208825084074}58052814\ldots$$

$$\Delta_{220}(\mathbf{427.208825084075}) = +9.85564\ldots\cdot 10^{-17794} > 0$$

Blowing the trumpet: the outcome

$$\Delta_{220}(\mathbf{427.208825084074}) = -1.92776\ldots \cdot 10^{-17793} < 0$$

$$\gamma_{220} = \mathbf{427.208825084074}58052814\ldots$$

$$\Delta_{220}(\mathbf{427.208825084075}) = +9.85564\ldots \cdot 10^{-17794} > 0$$

$$\Delta_{220}(\mathbf{428.127914076616}) = +3.30722\ldots \cdot 10^{-17792} > 0$$

$$\gamma_{221} = \mathbf{428.127914076616}68211030\ldots$$

$$\Delta_{220}(\mathbf{428.127914076617}) = -1.28498\ldots \cdot 10^{-17792} < 0$$

Blowing the trumpet: the outcome

$$\Delta_{220}(\mathbf{427.208825084074}) = -1.92776\ldots \cdot 10^{-17793} < 0$$
$$\gamma_{220} = \mathbf{427.208825084074}58052814\ldots$$
$$\Delta_{220}(\mathbf{427.208825084075}) = +9.85564\ldots \cdot 10^{-17794} > 0$$

$$\Delta_{220}(\mathbf{428.127914076616}) = +3.30722\ldots \cdot 10^{-17792} > 0$$
$$\gamma_{221} = \mathbf{428.127914076616}68211030\ldots$$
$$\Delta_{220}(\mathbf{428.127914076617}) = -1.28498\ldots \cdot 10^{-17792} < 0$$

$$\Delta_{220}(\mathbf{430.3287454309386}) = -1.08026\ldots \cdot 10^{-17794} < 0$$
$$\gamma_{222} = \mathbf{430.3287454309386}36669926\ldots$$
$$\Delta_{220}(\mathbf{430.3287454309387}) = +1.56602\ldots \cdot 10^{-17793} > 0$$

Blowing the trumpet: the outcome

$$\Delta_{220}(\mathbf{427.208825084074}) = -1.92776\ldots \cdot 10^{-17793} < 0$$
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$$\Delta_{220}(\mathbf{427.208825084075}) = +9.85564\ldots \cdot 10^{-17794} > 0$$

$$\Delta_{220}(\mathbf{428.127914076616}) = +3.30722\ldots \cdot 10^{-17792} > 0$$
$$\gamma_{221} = \mathbf{428.127914076616}68211030\ldots$$
$$\Delta_{220}(\mathbf{428.127914076617}) = -1.28498\ldots \cdot 10^{-17792} < 0$$

$$\Delta_{220}(\mathbf{430.3287454309386}) = -1.08026\ldots \cdot 10^{-17794} < 0$$
$$\gamma_{222} = \mathbf{430.3287454309386}36669926\ldots$$
$$\Delta_{220}(\mathbf{430.3287454309387}) = +1.56602\ldots \cdot 10^{-17793} > 0$$

. . . .

$$\Delta_{220}(\mathbf{441.683199201}) = -3.85957\ldots \cdot 10^{-17794} < 0$$
$$\gamma_{230} = \mathbf{441.683199201}18902387\ldots$$
$$\Delta_{220}(\mathbf{441.683199202}) = +1.39118\ldots \cdot 10^{-17793} > 0$$

Blowing the trumpet: the outcome

$\Delta_{12000}(t)$ has zeroes having more than 2000 common decimal digits with $\gamma_{12000},\ \gamma_{12001},\dots,\gamma_{12010}$

Partial explanation

$$\Xi(t) = \sum_{n=1}^{\infty} \beta_n(t)$$

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$$\Delta_N(t) = \begin{vmatrix} \beta_1(\gamma_1) & \dots & \beta_1(\gamma_{N-1}) & \beta_1(t) \\ \vdots & \ddots & \vdots & \vdots \\ \beta_N(\gamma_1) & \dots & \beta_N(\gamma_{N-1}) & \beta_N(t) \end{vmatrix}$$

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$$\tilde{\delta}_{N,n} = (-1)^{N+n} \begin{vmatrix} \beta_1(\gamma_1) & \dots & \beta_1(\gamma_{N-1}) \\ \vdots & \ddots & \vdots \\ \beta_{n-1}(\gamma_1) & \dots & \beta_{n-1}(\gamma_{N-1}) \\ \beta_{n+1}(\gamma_1) & \dots & \beta_{n+1}(\gamma_{N-1}) \\ \vdots & \ddots & \vdots \\ \beta_N(\gamma_1) & \dots & \beta_N(\gamma_{N-1}) \end{vmatrix}$$

Normalization

$$\Xi(t) = \sum_{n=1}^{\infty} \beta_n(t)$$

$$\Delta_N(t) = \begin{vmatrix} \beta_1(\gamma_1) & \dots & \beta_1(\gamma_{N-1}) & \beta_1(t) \\ \vdots & \ddots & \vdots & \vdots \\ \beta_N(\gamma_1) & \dots & \beta_N(\gamma_{N-1}) & \beta_N(t) \end{vmatrix} = \sum_{n=1}^{N} \tilde{\delta}_{N,n} \beta_n(t)$$

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$$\delta_{N,n} = \frac{\tilde{\delta}_{N,n}}{\tilde{\delta}_{N,1}}$$

Normalization

$$\Xi(t) = \sum_{n=1}^{\infty} \beta_n(t)$$

$$\Delta_N(t) = \begin{vmatrix} \beta_1(\gamma_1) & \dots & \beta_1(\gamma_{N-1}) & \beta_1(t) \\ \vdots & \ddots & \vdots & \vdots \\ \beta_N(\gamma_1) & \dots & \beta_N(\gamma_{N-1}) & \beta_N(t) \end{vmatrix} = \sum_{n=1}^{N} \tilde{\delta}_{N,n} \beta_n(t)$$

$$\delta_{N,n} = \frac{\tilde{\delta}_{N,n}}{\tilde{\delta}_{N,1}} \qquad \delta_{N,1} = 1$$

Normalization

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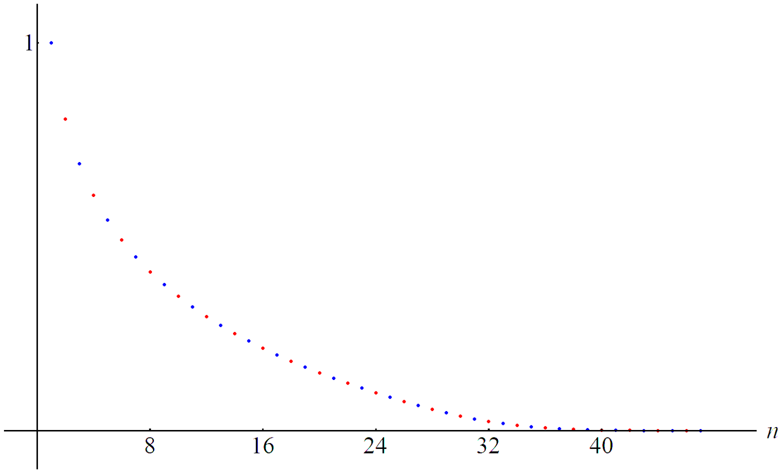
$$\Delta_N(t) = \begin{vmatrix} \beta_1(\gamma_1) & \dots & \beta_1(\gamma_{N-1}) & \beta_1(t) \\ \vdots & \ddots & \vdots & \vdots \\ \beta_N(\gamma_1) & \dots & \beta_N(\gamma_{N-1}) & \beta_N(t) \end{vmatrix} = \sum_{n=1}^{N} \tilde{\delta}_{N,n} \beta_n(t)$$

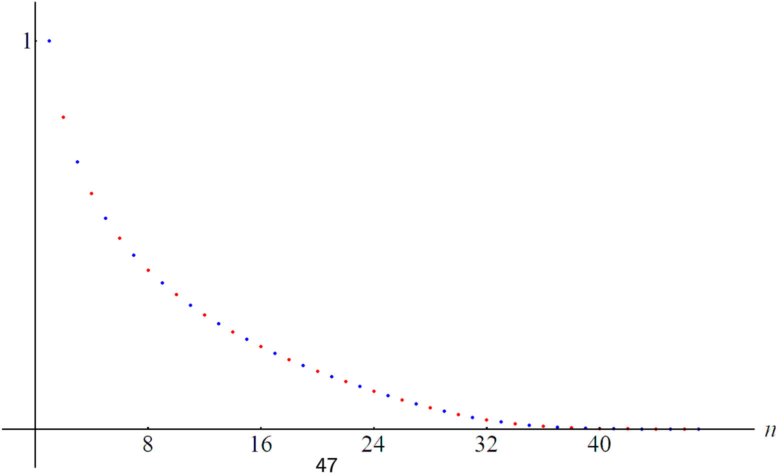
$$\delta_{N,n} = \frac{\tilde{\delta}_{N,n}}{\tilde{\delta}_{N,1}} \qquad \delta_{N,1} = 1$$

$$\sum_{n=1}^{N} \delta_{N,n} \beta_n(t)$$

Normalized coefficients $\delta_{47,n}$

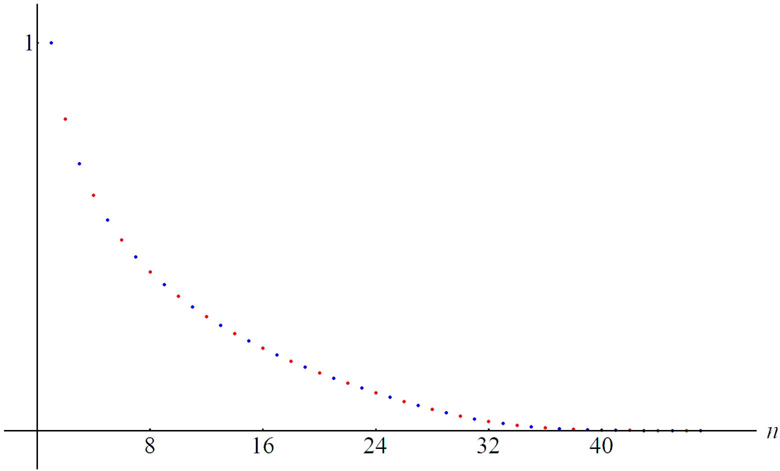


Normalized coefficients $\delta_{47,n}$

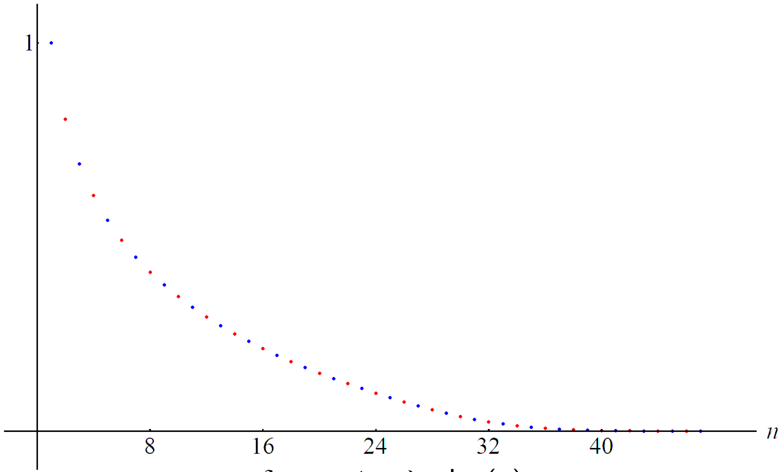


$$\sum_{n=1}^{47} \delta_{47,n}\beta_n(t)$$

Normalized coefficients $\delta_{47,n}$



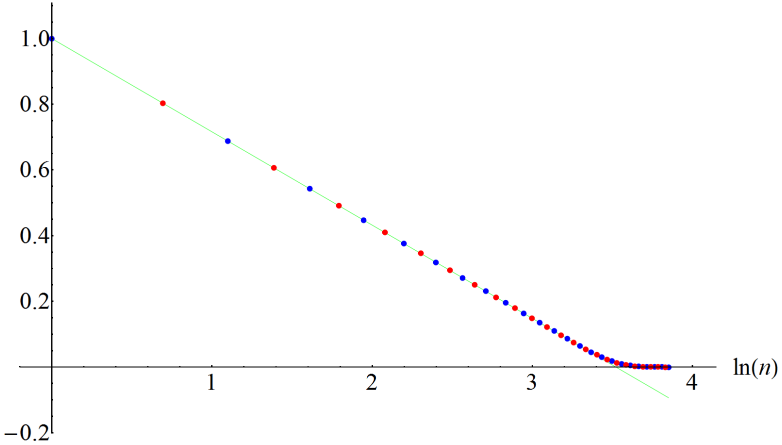
Normalized coefficients $\delta_{47,n}$



$$\delta_{47,n} \approx 1 + \lambda_{47} \log(n)$$

Normalized coefficients $\delta_{47,n}$ with logarithmic scale

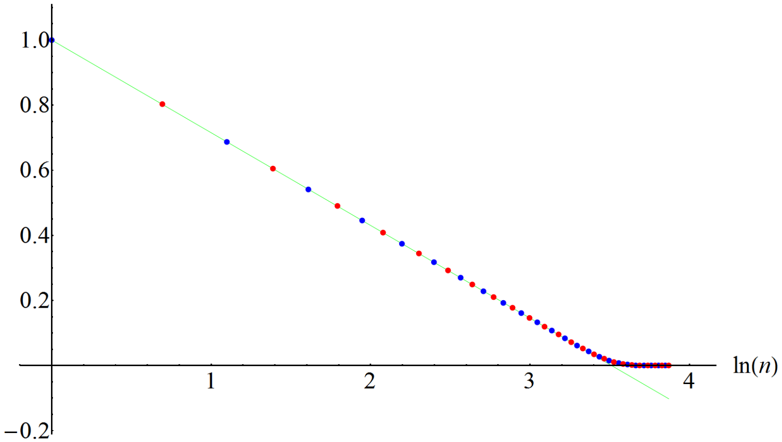
Normalized coefficients $\delta_{47,n}$ with logarithmic scale



$$\delta_{47,n} \approx 1 + \lambda_{47} \log(n)$$

Normalized coefficients $\delta_{48,n}$ with logarithmic scale

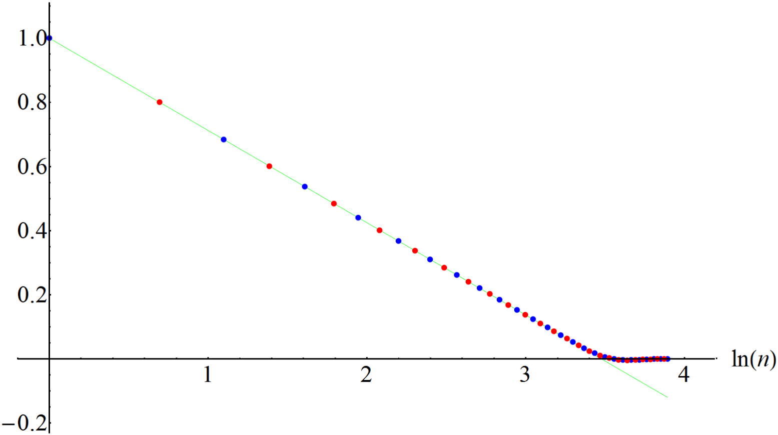
Normalized coefficients $\delta_{48,n}$ with logarithmic scale



$$\delta_{48,n} \approx 1 + \lambda_{48} \log(n)$$

Normalized coefficients $\delta_{49,n}$ with logarithmic scale

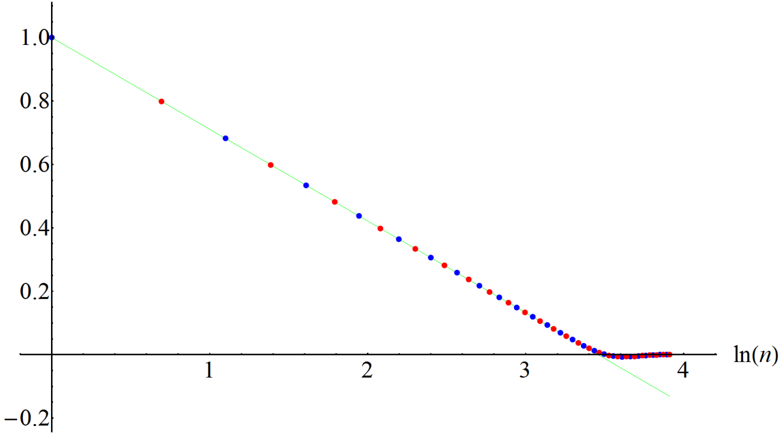
Normalized coefficients $\delta_{49,n}$ with logarithmic scale



$$\delta_{49,n} \approx 1 + \lambda_{49} \log(n)$$

Normalized coefficients $\delta_{50,n}$ with logarithmic scale

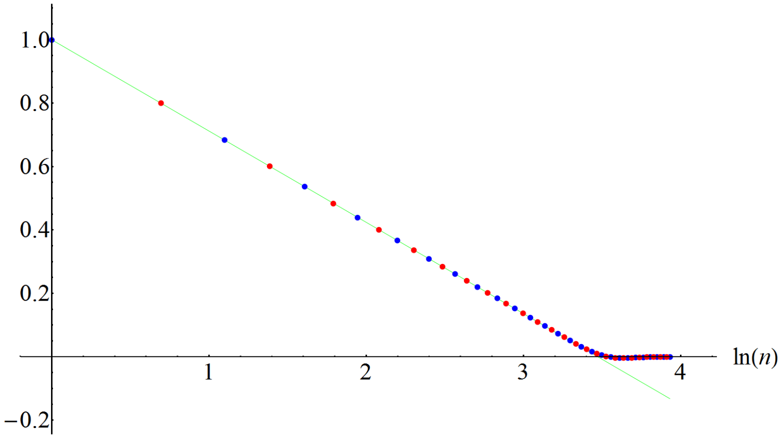
Normalized coefficients $\delta_{50,n}$ with logarithmic scale



$$\delta_{50,n} \approx 1 + \lambda_{50} \log(n)$$

Normalized coefficients $\delta_{51,n}$ with logarithmic scale

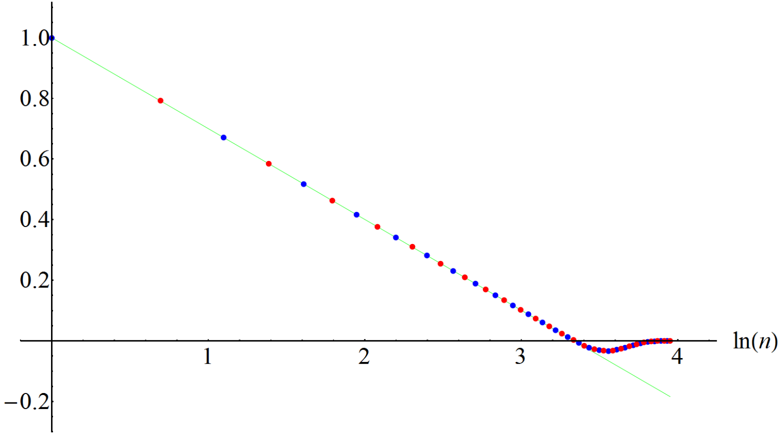
Normalized coefficients $\delta_{51,n}$ with logarithmic scale



$$\delta_{51,n} \approx 1 + \lambda_{52} \log(n)$$

Normalized coefficients $\delta_{52,n}$ with logarithmic scale

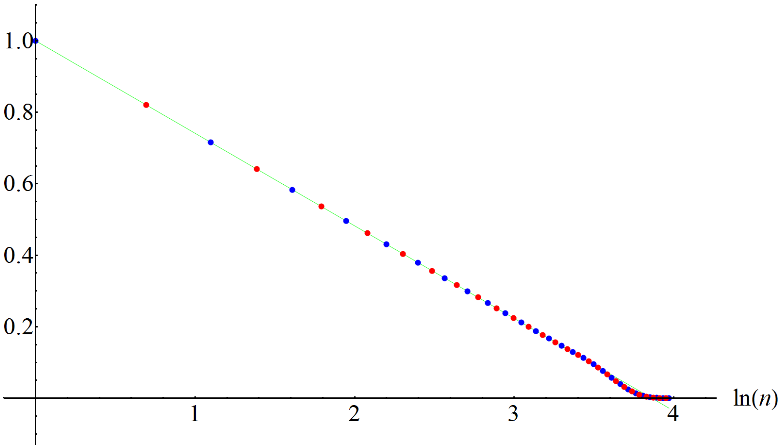
Normalized coefficients $\delta_{52,n}$ with logarithmic scale



$$\delta_{52,n} \approx 1 + \lambda_{52} \log(n)$$

Normalized coefficients $\delta_{53,n}$ with logarithmic scale

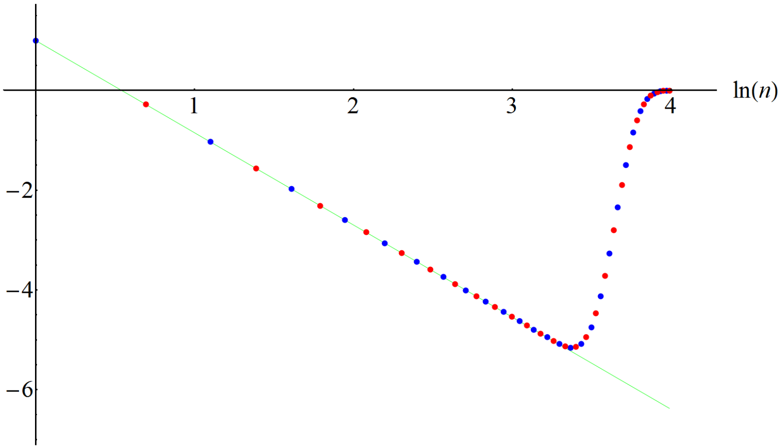
Normalized coefficients $\delta_{53,n}$ with logarithmic scale



$$\delta_{53,n} \approx 1 + \lambda_{53} \log(n)$$

Normalized coefficients $\delta_{54,n}$ with logarithmic scale

Normalized coefficients $\delta_{54,n}$ with logarithmic scale



$$\delta_{54,n} \approx 1 + \lambda_{54} \log(n)$$

Normalized coefficients $\delta_{55,n}$ with logarithmic scale

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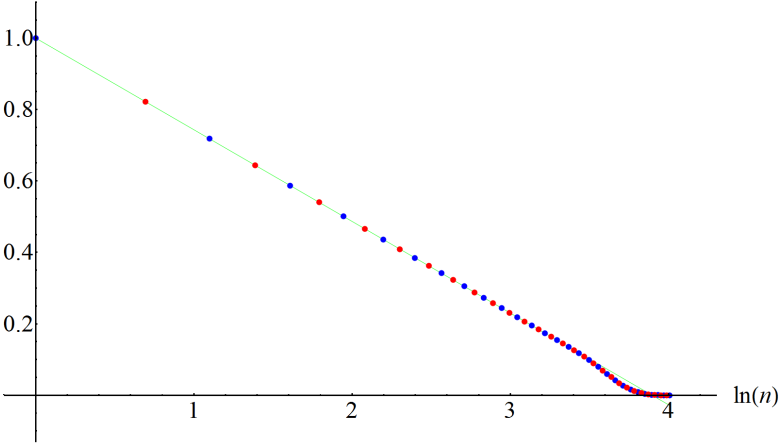
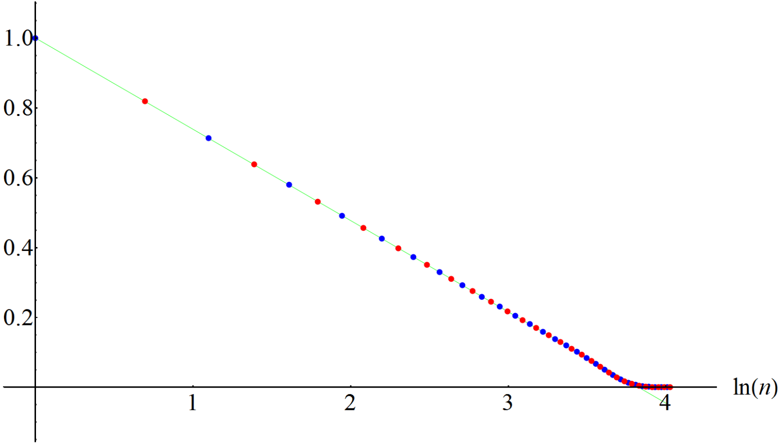


$$\delta_{55,n} \approx 1 + \lambda_{55} \log(n)$$

Normalized coefficients $\delta_{56,n}$ with logarithmic scale

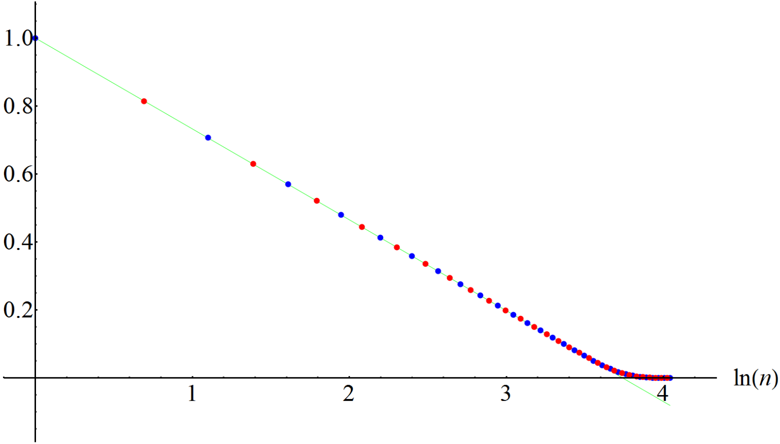
Normalized coefficients $\delta_{56,n}$ with logarithmic scale



$$\delta_{56,n} \approx 1 + \lambda_{56} \log(n)$$

Normalized coefficients $\delta_{57,n}$ with logarithmic scale

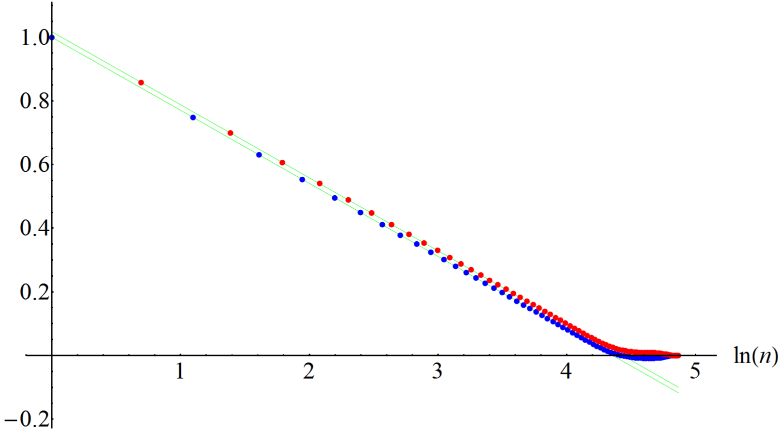
Normalized coefficients $\delta_{57,n}$ with logarithmic scale



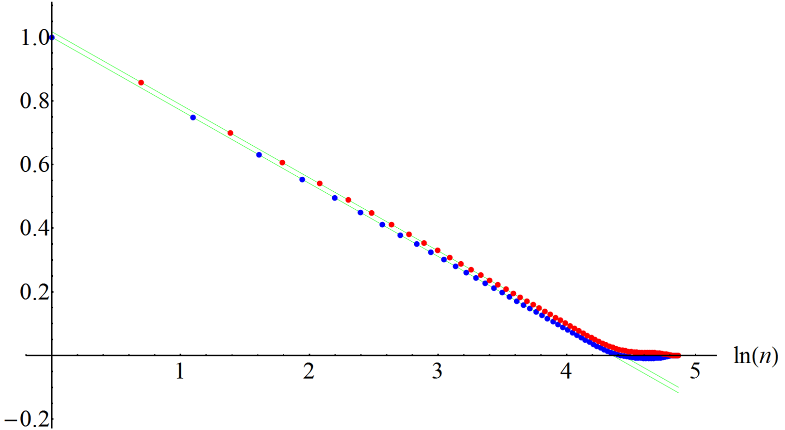
$$\delta_{57,n} \approx 1 + \lambda_{57} \log(n)$$

Normalized coefficients $\delta_{130,n}$ with logarithmic scale

Normalized coefficients $\delta_{130,n}$ with logarithmic scale



Normalized coefficients $\delta_{130,n}$ with logarithmic scale

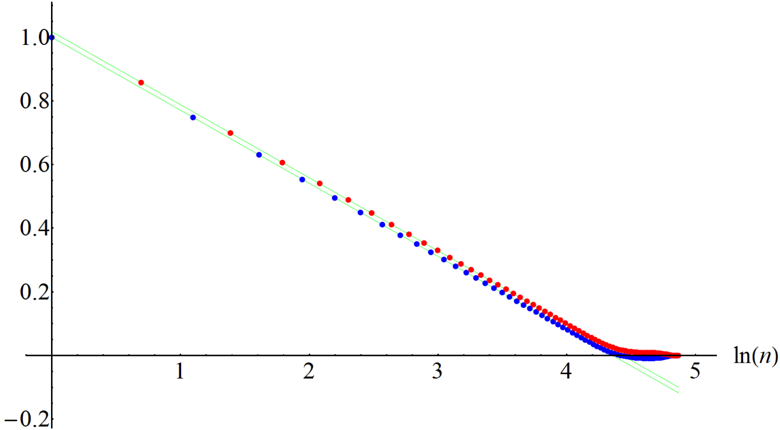


$$\delta_{130,n} \approx 1 + \mu_{130,2}\mathrm{dom}_2(n) + \lambda_{130}\log(n)$$

The characteristic function of the divisibility

$$\mathrm{dom}_m(k) = \begin{cases} 1, & \text{if } m \mid k \\ 0, & \text{otherwise} \end{cases}$$

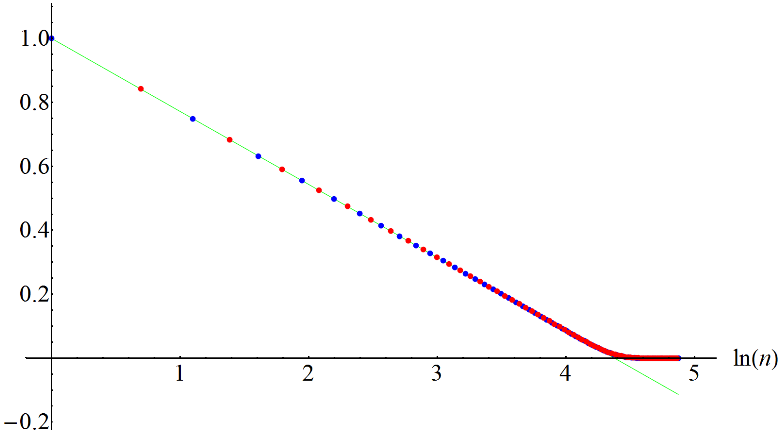
Normalized coefficients $\delta_{130,n}$ with logarithmic scale



$$\delta_{130,n} \approx 1 + \mu_{130,2}\mathrm{dom}_2(n) + \lambda_{130}\log(n)$$

Normalized coefficients $\delta_{131,n}$ with logarithmic scale

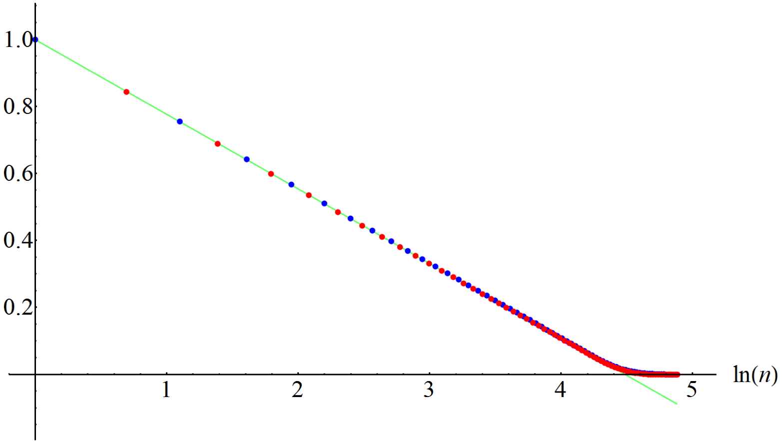
Normalized coefficients $\delta_{131,n}$ with logarithmic scale



$$\delta_{131,n} \approx 1 + \mu_{131,2}\mathrm{dom}_2(n) + \lambda_{131}\log(n)$$

Normalized coefficients $\delta_{132,n}$ with logarithmic scale

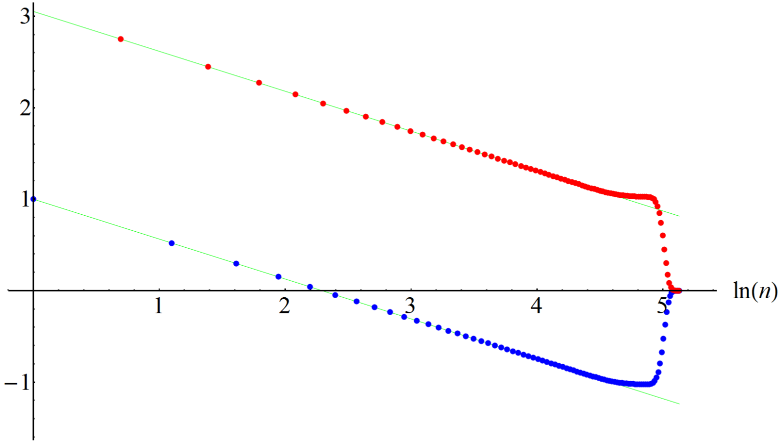
Normalized coefficients $\delta_{132,n}$ with logarithmic scale



$$\delta_{132,n} \approx 1 + \mu_{132,2}\mathrm{dom}_2(n) + \lambda_{132}\log(n)$$

Normalized coefficients $\delta_{169,n}$ with logarithmic scale

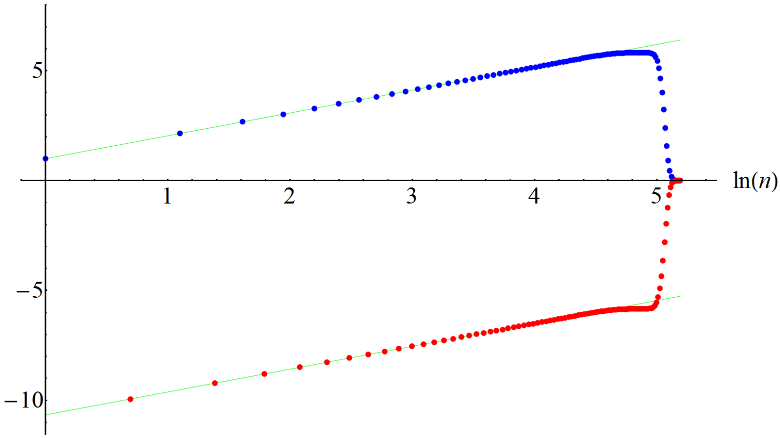
Normalized coefficients $\delta_{169,n}$ with logarithmic scale



$$\delta_{169,n} \approx 1 + \mu_{169,2}\mathrm{dom}_2(n) + \lambda_{169}\log(n)$$

Normalized coefficients $\delta_{180,n}$ with logarithmic scale

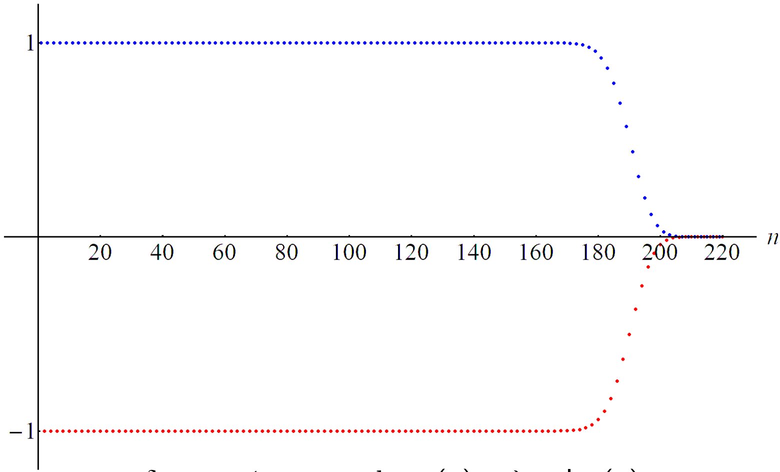
Normalized coefficients $\delta_{180,n}$ with logarithmic scale



$$\delta_{180,n} \approx 1 + \mu_{180,2}\mathrm{dom}_2(n) + \lambda_{180}\log(n)$$

Normalized coefficients $\delta_{220,n}$

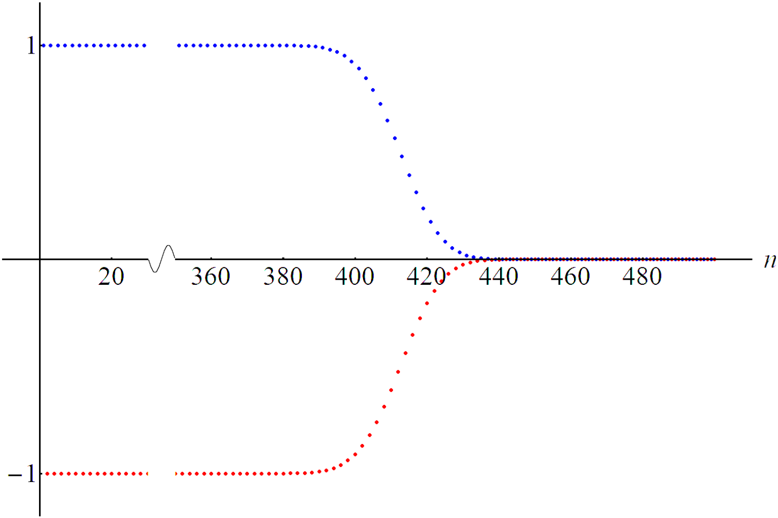
Normalized coefficients $\delta_{220,n}$



$$\delta_{220,n} \approx 1 + \mu_{220,2}\mathrm{dom}_2(n) + \lambda_{220}\log(n)$$

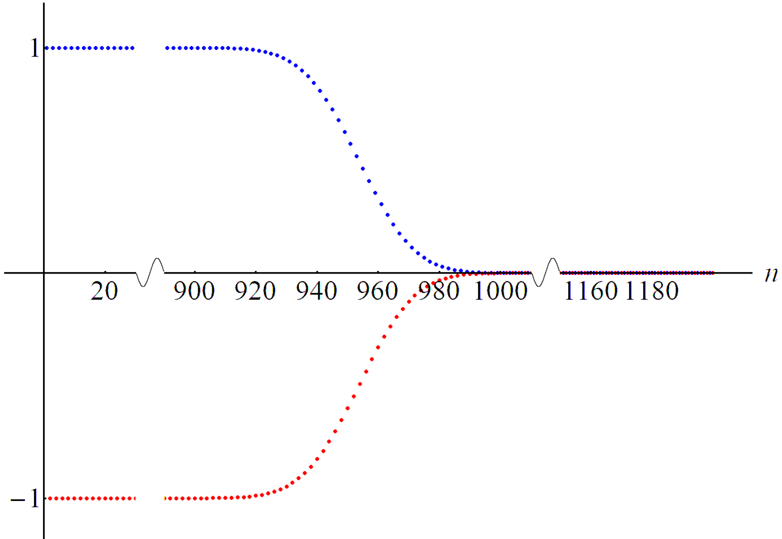
Normalized coefficients $\delta_{500,n}$

Normalized coefficients $\delta_{500,n}$



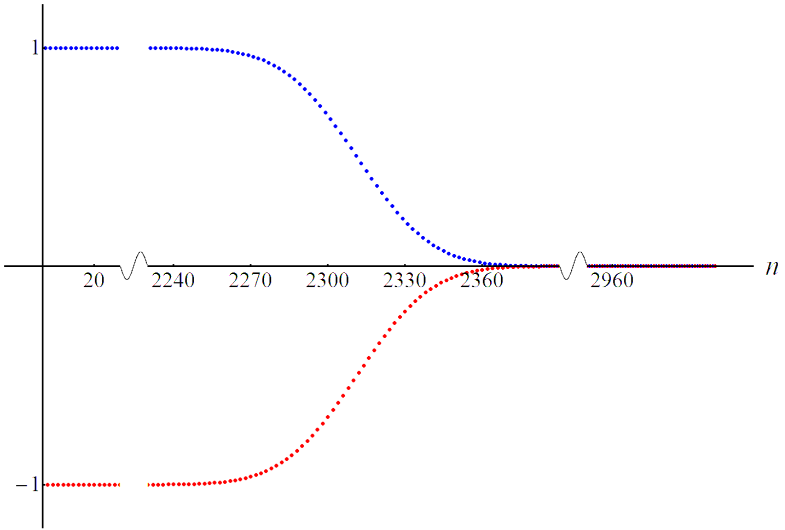
Normalized coefficients $\delta_{1200,n}$

Normalized coefficients $\delta_{1200,n}$



Normalized coefficients $\delta_{3000,n}$

Normalized coefficients $\delta_{3000,n}$



Partial explanation

The function $\Delta_{220}(t)$ is a "smooth" truncation, not of the divergent Diriclet series

$$\Xi(t) = \sum_{n=1}^{\infty} \beta_n(t) = \sum_{n=1}^{\infty} \frac{\alpha_n(t) + \alpha_n(-t)}{2}$$

Partial explanation

The function $\Delta_{220}(t)$ is a "smooth" truncation, not of the divergent Diriclet series

$$\Xi(t)=\sum_{n=1}^{\infty}\beta_n(t)=\sum_{n=1}^{\infty}\frac{\alpha_n(t)+\alpha_n(-t)}{2}$$

but of the convergent (for real t) alternating series

$$\sum_{n=1}^{\infty}(-1)^{n-1}\beta_n(t)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\alpha_n(t)+\alpha_n(-t)}{2}$$

Partial explanation

The function $\Delta_{220}(t)$ is a "smooth" truncation, not of the divergent Diriclet series

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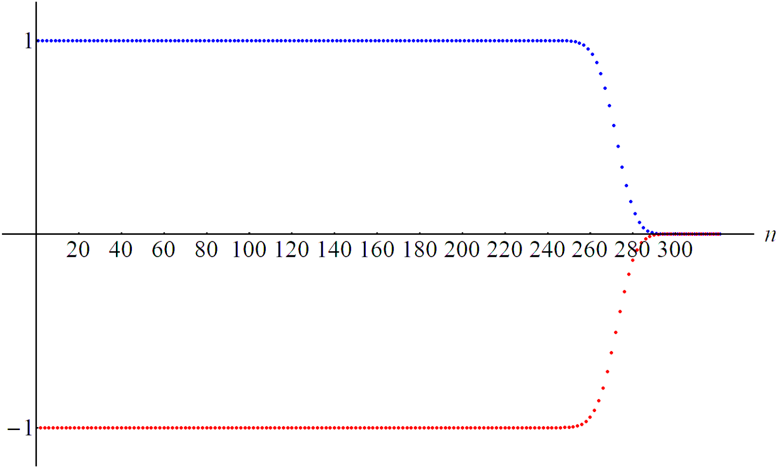
but of the convergent (for real t) alternating series

$$\begin{aligned}
\sum_{n=1}^{\infty}(-1)^{n-1}\beta_n(t) &= \sum_{n=1}^{\infty}(-1)^{n-1}\frac{\alpha_n(t) + \alpha_n(-t)}{2} \\
&= \sum_{n=1}^{\infty}(-1)^{n-1}\left(t^2 + \tfrac{1}{4}\right)\left(\frac{\pi^{-\frac{1}{4}+\frac{\mathrm{i}t}{2}}\Gamma\left(\frac{1}{4} - \frac{\mathrm{i}t}{2}\right)}{4n^{\frac{1}{2}-\mathrm{i}t}} + \frac{\pi^{-\frac{1}{4}-\frac{\mathrm{i}t}{2}}\Gamma\left(\frac{1}{4} + \frac{\mathrm{i}t}{2}\right)}{4n^{\frac{1}{2}+\mathrm{i}t}}\right) \\
&= \frac{1}{4}\left(t^2 + \tfrac{1}{4}\right)\pi^{-\frac{1}{4}+\frac{\mathrm{i}t}{2}}\Gamma\left(\tfrac{1}{4} - \tfrac{\mathrm{i}t}{2}\right)\sum_{n=1}^{\infty}(-1)^{n-1}n^{-\frac{1}{2}+\mathrm{i}t} + \\
&\quad \frac{1}{4}\left(t^2 + \tfrac{1}{4}\right)\pi^{-\frac{1}{4}-\frac{\mathrm{i}t}{2}}\Gamma\left(\tfrac{1}{4} + \tfrac{\mathrm{i}t}{2}\right)\sum_{n=1}^{\infty}(-1)^{n-1}n^{-\frac{1}{2}-\mathrm{i}t}
\end{aligned}$$

Normalized coefficients $\delta_{321,n}$



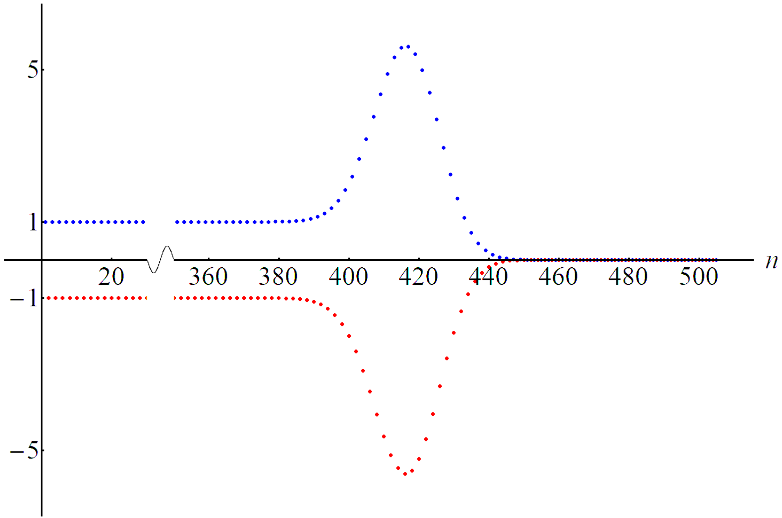
A Conjecture

A Conjecture. *For every real* ν *such that* $1 \leq \nu$ *the sequence*

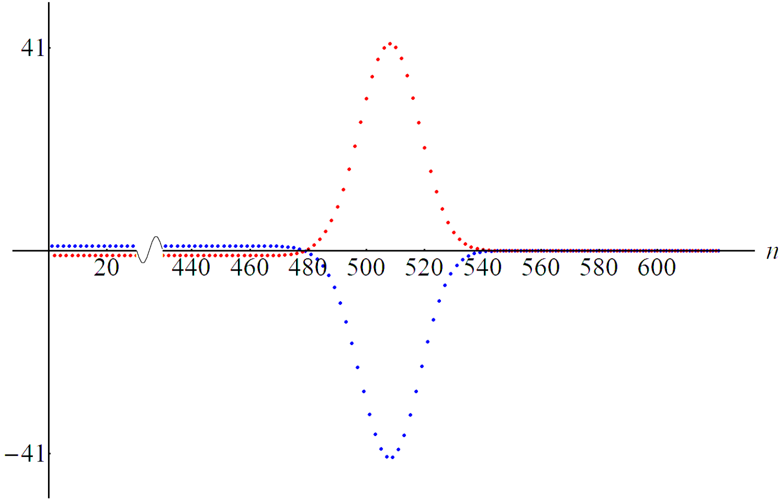
$$\delta_{\lfloor \nu \rfloor, 1}, \ldots, (-1)^{n-1} \delta_{\lfloor \nu n \rfloor, n}, \ldots$$

has certain limiting value $\delta(\nu)$*.*

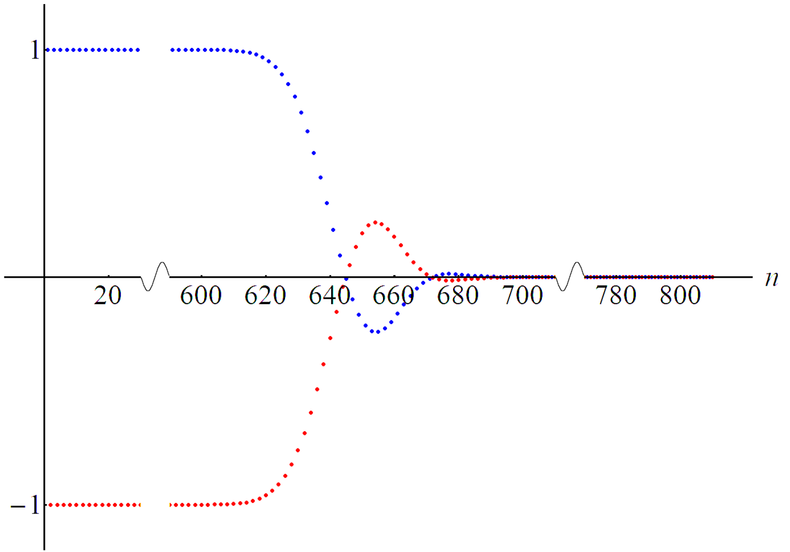
Normalized coefficients $\delta_{505,n}$



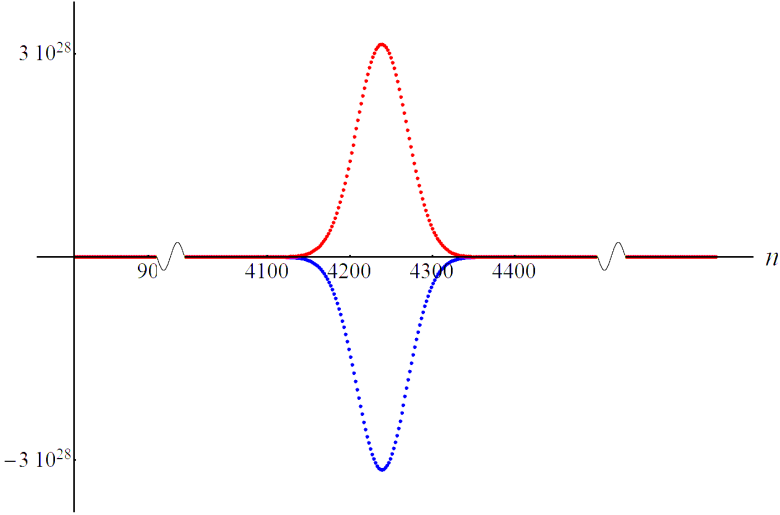
Normalized coefficients $\delta_{621,n}$



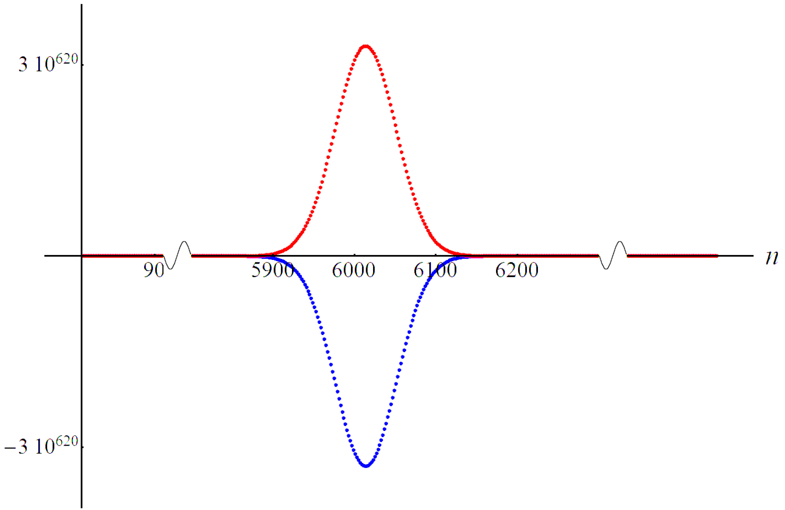
Normalized coefficients $\delta_{810,n}$



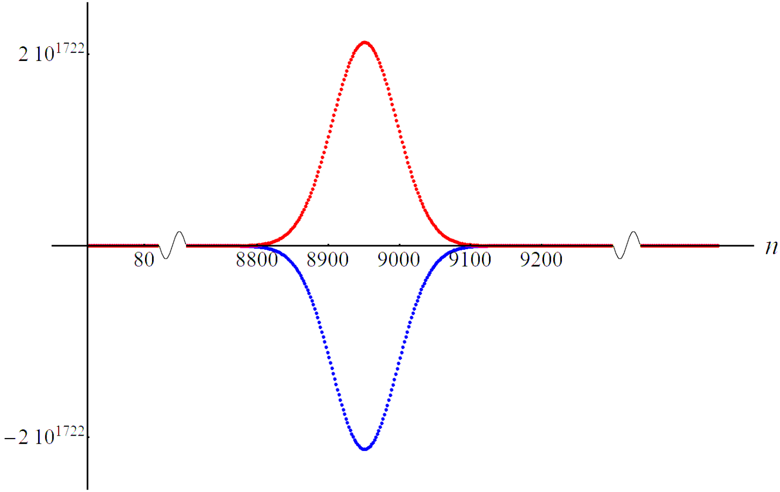
Normalized coefficients $\delta_{5600,n}$



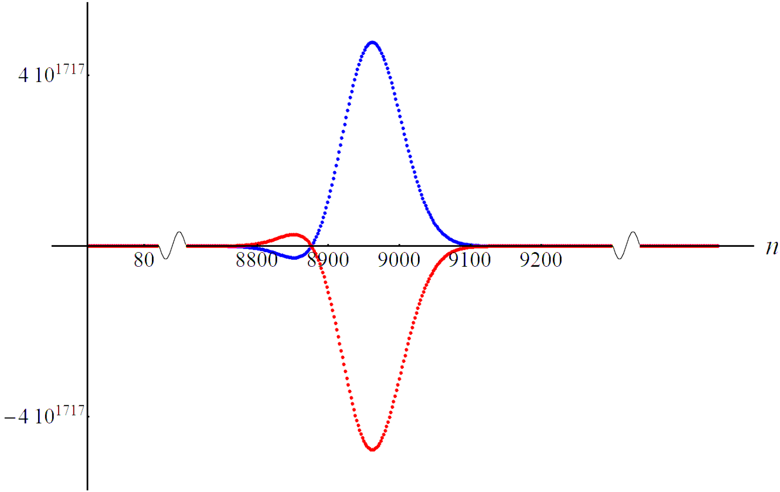
Normalized coefficients $\delta_{8000,n}$



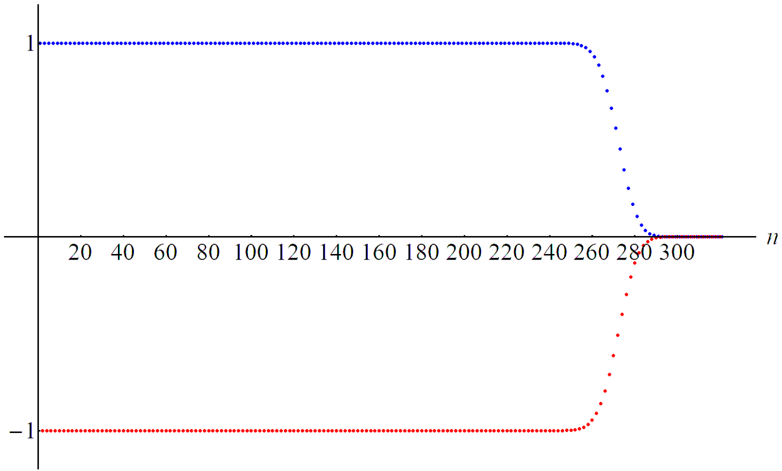
Normalized coefficients $\delta_{12000,n}$



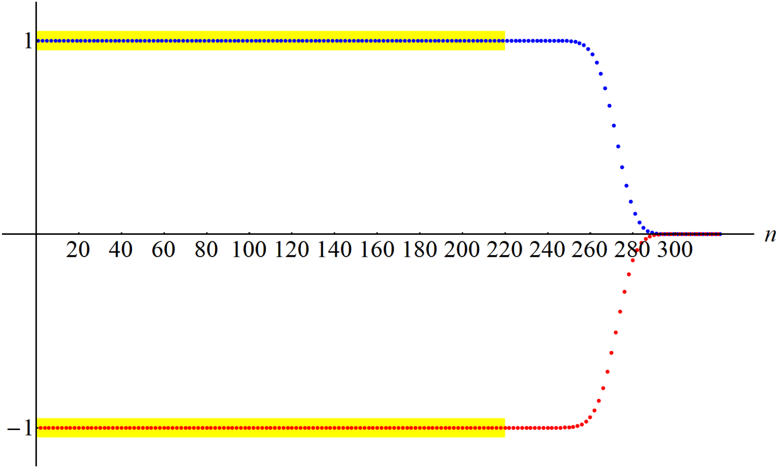
Normalized coefficients $\delta_{11981,n}$



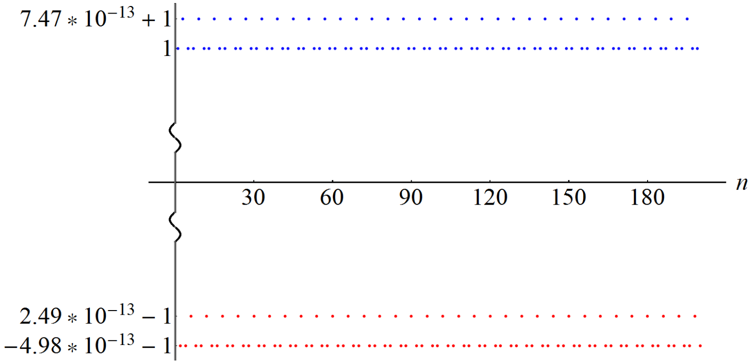
Normalized coefficients $\delta_{321,n}$



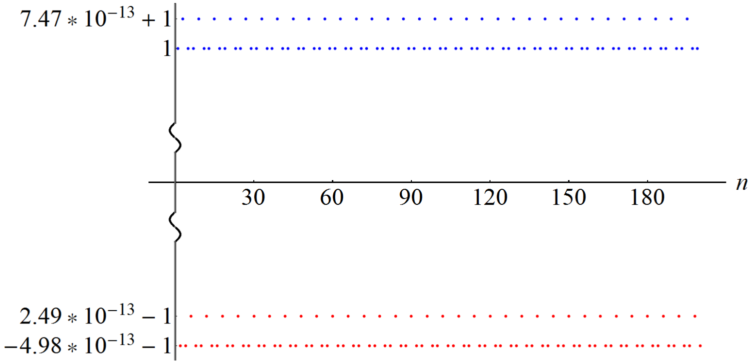
Normalized coefficients $\delta_{321,n}$



Normalized coefficients $\delta_{321,n}$

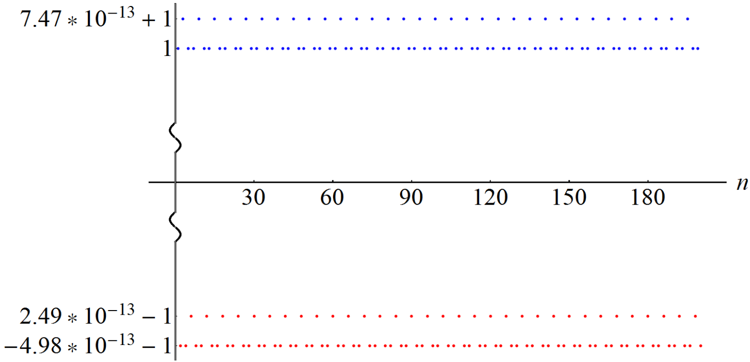


Normalized coefficients $\delta_{321,n}$



$$\delta_{321,n} \approx 1 + \mu_{321,2}\mathrm{dom}_2(n) + \mu_{321,3}\mathrm{dom}_3(n) + \lambda_{321}\log(n)$$

Normalized coefficients $\delta_{321,n}$



$$\delta_{321,n} \approx 1 + \mu_{321,2}\mathrm{dom}_2(n) + \mu_{321,3}\mathrm{dom}_3(n) + \lambda_{321}\log(n)$$

$$\mu_{321,2} = -2 - 4.98\ldots \cdot 10^{-13} \qquad \mu_{321,3} = 7.47\ldots \cdot 10^{-13}$$

$$\lambda_{321} = -3.33\ldots \cdot 10^{-18}$$

Normalized coefficients: general case

Normalized coefficients: general case

$$\delta_{N,n} \approx \sum_m \mu_{N,m} \mathrm{dom}_m(n) + \lambda_N \log(n)$$

Normalized coefficients: general case

$$\delta_{N,n} \approx \sum_m \mu_{N,m} \mathrm{dom}_m(n) + \lambda_N \log(n)$$

$$\mu_{N,1} = 1 \qquad \mathrm{dom}_1(m) \equiv 1$$

Averaged normalized coefficients

$$\delta_{N,n,a} = \frac{\delta_{N,n} + \cdots + \delta_{N,n+a-1}}{a}$$

Averaged normalized coefficients

$$\delta_{N,n,a} = \frac{\delta_{N,n} + \cdots + \delta_{N,n+a-1}}{a}$$

$$\delta_{N,n,2} = \frac{\delta_{N,n} + \delta_{N,n+1}}{2}$$

Averaged normalized coefficients

$$\delta_{N,n,a} = \frac{\delta_{N,n} + \cdots + \delta_{N,n+a-1}}{a}$$

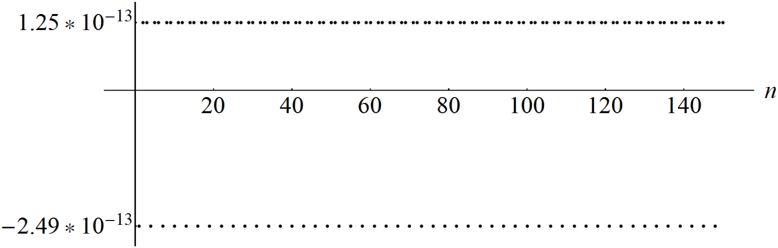
$$\begin{aligned}
\delta_{N,n,2} &= \frac{\delta_{N,n} + \delta_{N,n+1}}{2} \\
&\approx 1 + \mu_{N,2} \frac{\mathrm{dom}_2(n) + \mathrm{dom}_2(n+1)}{2} + \\
&\quad \mu_{N,3} \frac{\mathrm{dom}_3(n) + \mathrm{dom}_3(n+1)}{2} + \lambda_N \frac{\log(n(n+1))}{2}
\end{aligned}$$

Averaged normalized coefficients

$$\delta_{N,n,a} = \frac{\delta_{N,n} + \cdots + \delta_{N,n+a-1}}{a}$$

$$\begin{aligned}
\delta_{N,n,2} &= \frac{\delta_{N,n} + \delta_{N,n+1}}{2} \\
&\approx 1 + \mu_{N,2} \frac{\mathrm{dom}_2(n) + \mathrm{dom}_2(n+1)}{2} + \\
&\quad \mu_{N,3} \frac{\mathrm{dom}_3(n) + \mathrm{dom}_3(n+1)}{2} + \lambda_N \frac{\log(n(n+1))}{2} \\
&= 1 + \frac{\mu_{N,2}}{2} + \\
&\quad \mu_{N,3} \frac{\mathrm{dom}_3(n) + \mathrm{dom}_3(n+1)}{2} + \lambda_N \frac{\log(n(n+1))}{2}
\end{aligned}$$

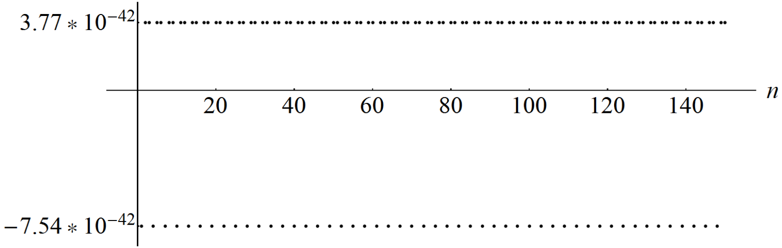
Averaged normalized coefficients $\delta_{321,n,2}$



$$\mu_{321,3} = 7.47\ldots \cdot 10^{-13}$$

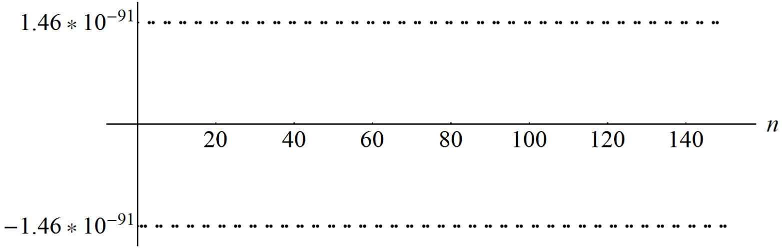
Averaged normalized coefficients $\delta_{999,n,2}$

Averaged normalized coefficients $\delta_{999,n,2}$



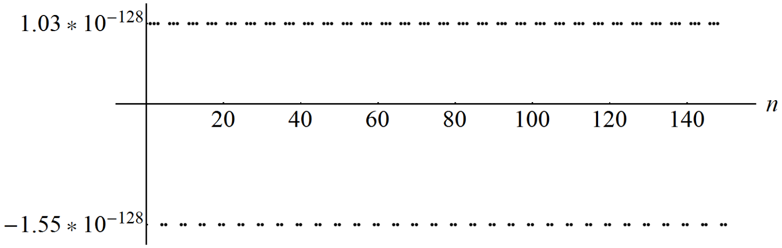
$$\mu_{999,3} = 2.26\ldots \cdot 10^{-41}$$

Averaged normalized coefficients $\delta_{999,n,6}$



$$\mu_{999,4} = 1.75\ldots \cdot 10^{-90}$$

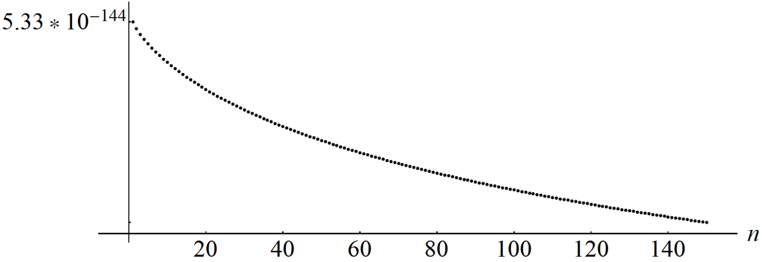
Averaged normalized coefficients $\delta_{999,n,12}$



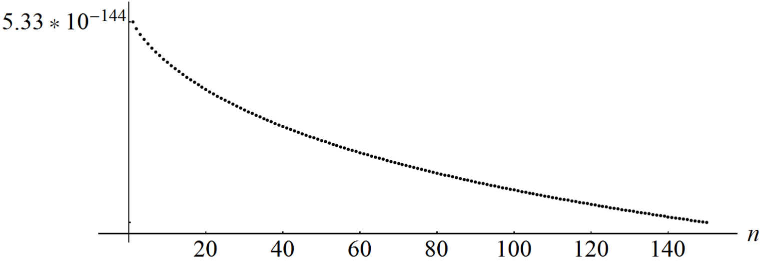
$$\mu_{999,5} = -3.09\ldots \cdot 10^{-127}$$

Averaged normalized coefficients $\delta_{999,n,60}$

Averaged normalized coefficients $\delta_{999,n,60}$

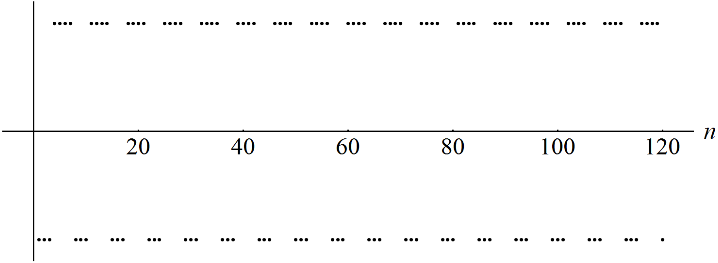


Averaged normalized coefficients $\delta_{999,n,60}$



$$\lambda_{999} = -1.73\ldots \cdot 10^{-144}$$

Differences $\delta_{999,n,60} - \lambda_{999} \log(\Gamma(n+60) - \Gamma(n))/60$



$$\lambda_{999} = -1.73\ldots \cdot 10^{-144} \qquad \mu_{999,7} = 1.52\ldots \cdot 10^{-171}$$

Almost linear relations

Almost linear relations

$$r_1\delta_{N,1} + r_2\delta_{N,2} + \cdots + r_m\delta_{N,m} = r$$

$r_1, r_2, \ldots, r_m$, and r are either rational numbers with small denominators or very close to integers

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1}$ $= -2.00... \cdot 10^{0}$	$2 + 3.00... \cdot 10^{-134}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1}$ $= -2.00... \cdot 10^{0}$	$2 + 3.00... \cdot 10^{-134}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = -2.00... \cdot 10^{0}$	$2 + 3.00... \cdot 10^{-134}$
3	$-1.50... \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50... \cdot 10^{-134}$	$3 - 5.41... \cdot 10^{-171}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

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2	1	$\delta_{N,2} - \mu_{N,1}$ $= -2.00... \cdot 10^{0}$	$2 + 3.00... \cdot 10^{-134}$
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n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = -2.00\ldots \cdot 10^{0}$	$2 + 3.00\ldots \cdot 10^{-134}$
3	$-1.50\ldots \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50\ldots \cdot 10^{-134}$	$3 - 5.41\ldots \cdot 10^{-171}$
4	$-2.71\ldots \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} = -2.00\ldots \cdot 10^{0}$	$-7.37\ldots \cdot 10^{304}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

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2	1	$\delta_{N,2} - \mu_{N,1} = -2.00... \cdot 10^{0}$	$2 + 3.00... \cdot 10^{-134}$
3	$-1.50... \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50... \cdot 10^{-134}$	$3 - 5.41... \cdot 10^{-171}$
4	$-2.71... \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} = -2.00... \cdot 10^{0}$	$-7.37... \cdot 10^{304}$

$$\frac{\mu_{N,2}}{\mu_{N,4}} = 1 + 5.42... \cdot 10^{-305}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1}$ $= -2.00... \cdot 10^{0}$	$2 + 3.00... \cdot 10^{-134}$
3	$-1.50... \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1}$ $= +4.50... \cdot 10^{-134}$	$3 - 5.41... \cdot 10^{-171}$
4	$-2.71... \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2}$ $= +1.08... \cdot 10^{-304}$	$4 - 6.96... \cdot 10^{-138}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

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2	1	$\delta_{N,2} - \mu_{N,1} = -2.00... \cdot 10^{0}$	$2 + 3.00... \cdot 10^{-134}$
3	$-1.50... \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50... \cdot 10^{-134}$	$3 - 5.41... \cdot 10^{-171}$
4	$-2.71... \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08... \cdot 10^{-304}$	$4 - 6.96... \cdot 10^{-138}$
5	$-4.72... \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36... \cdot 10^{-442}$	$5 - 3.14... \cdot 10^{-105}$

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3	$-1.50... \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1}$ $= +4.50... \cdot 10^{-134}$	$3 - 5.41... \cdot 10^{-171}$
4	$-2.71... \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2}$ $= +1.08... \cdot 10^{-304}$	$4 - 6.96... \cdot 10^{-138}$
5	$-4.72... \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1}$ $= +2.36... \cdot 10^{-442}$	$5 - 3.14... \cdot 10^{-105}$

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n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = -2.00\ldots \cdot 10^{0}$	$2 + 3.00\ldots \cdot 10^{-134}$
3	$-1.50\ldots \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50\ldots \cdot 10^{-134}$	$3 - 5.41\ldots \cdot 10^{-171}$
4	$-2.71\ldots \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08\ldots \cdot 10^{-304}$	$4 - 6.96\ldots \cdot 10^{-138}$
5	$-4.72\ldots \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36\ldots \cdot 10^{-442}$	$5 - 3.14\ldots \cdot 10^{-105}$
6	$-2.97\ldots \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} = -1.99\ldots \cdot 10^{0}$	$-6.73\ldots \cdot 10^{547}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1}$ $= -2.00\ldots \cdot 10^{0}$	$2 + 3.00\ldots \cdot 10^{-134}$
3	$-1.50\ldots \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1}$ $= +4.50\ldots \cdot 10^{-134}$	$3 - 5.41\ldots \cdot 10^{-171}$
4	$-2.71\ldots \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2}$ $= +1.08\ldots \cdot 10^{-304}$	$4 - 6.96\ldots \cdot 10^{-138}$
5	$-4.72\ldots \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1}$ $= +2.36\ldots \cdot 10^{-442}$	$5 - 3.14\ldots \cdot 10^{-105}$
6	$-2.97\ldots \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1}$ $= -1.99\ldots \cdot 10^{0}$	$-6.73\ldots \cdot 10^{547}$

$$\frac{\mu_{N,2}}{\mu_{N,6}} = 1 + 2.25\ldots \cdot 10^{-134}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = -2.00\ldots \cdot 10^{0}$	$2 + 3.00\ldots \cdot 10^{-134}$
3	$-1.50\ldots \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50\ldots \cdot 10^{-134}$	$3 - 5.41\ldots \cdot 10^{-171}$
4	$-2.71\ldots \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08\ldots \cdot 10^{-304}$	$4 - 6.96\ldots \cdot 10^{-138}$
5	$-4.72\ldots \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36\ldots \cdot 10^{-442}$	$5 - 3.14\ldots \cdot 10^{-105}$
6	$-2.97\ldots \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} = +4.50\ldots \cdot 10^{-134}$	$+1.51\ldots \cdot 10^{414}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = -2.00\ldots \cdot 10^{0}$	$2 + 3.00\ldots \cdot 10^{-134}$
3	$-1.50\ldots \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50\ldots \cdot 10^{-134}$	$3 - 5.41\ldots \cdot 10^{-171}$
4	$-2.71\ldots \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08\ldots \cdot 10^{-304}$	$4 - 6.96\ldots \cdot 10^{-138}$
5	$-4.72\ldots \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36\ldots \cdot 10^{-442}$	$5 - 3.14\ldots \cdot 10^{-105}$
6	$-2.97\ldots \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} = +4.50\ldots \cdot 10^{-134}$	$+1.51\ldots \cdot 10^{414}$

$$\frac{\mu_{N,3}}{\mu_{N,6}} = 1 - 3.95\ldots \cdot 10^{-414}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = -2.00... \cdot 10^{0}$	$2 + 3.00... \cdot 10^{-134}$
3	$-1.50... \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50... \cdot 10^{-134}$	$3 - 5.41... \cdot 10^{-171}$
4	$-2.71... \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08... \cdot 10^{-304}$	$4 - 6.96... \cdot 10^{-138}$
5	$-4.72... \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36... \cdot 10^{-442}$	$5 - 3.14... \cdot 10^{-105}$
6	$-2.97... \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.78... \cdot 10^{-547}$	$6 + 1.20... \cdot 10^{-81}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1}$ $= -2.00... \cdot 10^{0}$	$2 + 3.00... \cdot 10^{-134}$
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4	$-2.71... \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2}$ $= +1.08... \cdot 10^{-304}$	$4 - 6.96... \cdot 10^{-138}$
5	$-4.72... \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1}$ $= +2.36... \cdot 10^{-442}$	$5 - 3.14... \cdot 10^{-105}$
6	$-2.97... \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.78... \cdot 10^{-547}$	$6 + 1.20... \cdot 10^{-81}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = -2.00... \cdot 10^{0}$	$2 + 3.00... \cdot 10^{-134}$
3	$-1.50... \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50... \cdot 10^{-134}$	$3 - 5.41... \cdot 10^{-171}$
4	$-2.71... \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08... \cdot 10^{-304}$	$4 - 6.96... \cdot 10^{-138}$
5	$-4.72... \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36... \cdot 10^{-442}$	$5 - 3.14... \cdot 10^{-105}$
6	$-2.97... \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.78... \cdot 10^{-547}$	$6 + 1.20... \cdot 10^{-81}$
7	$+5.96... \cdot 10^{-630}$	$\delta_{N,7} - \mu_{N,1} = -4.17... \cdot 10^{-629}$	$7 + 1.08... \cdot 10^{-61}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = -2.00... \cdot 10^{0}$	$2 + 3.00... \cdot 10^{-134}$
3	$-1.50... \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50... \cdot 10^{-134}$	$3 - 5.41... \cdot 10^{-171}$
4	$-2.71... \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08... \cdot 10^{-304}$	$4 - 6.96... \cdot 10^{-138}$
5	$-4.72... \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36... \cdot 10^{-442}$	$5 - 3.14... \cdot 10^{-105}$
6	$-2.97... \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.78... \cdot 10^{-547}$	$6 + 1.20... \cdot 10^{-81}$
7	$+5.96... \cdot 10^{-630}$	$\delta_{N,7} - \mu_{N,1} = -4.17... \cdot 10^{-629}$	$7 + 1.08... \cdot 10^{-61}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = -2.00\ldots \cdot 10^{0}$	$2 + 3.00\ldots \cdot 10^{-134}$
3	$-1.50\ldots \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50\ldots \cdot 10^{-134}$	$3 - 5.41\ldots \cdot 10^{-171}$
4	$-2.71\ldots \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08\ldots \cdot 10^{-304}$	$4 - 6.96\ldots \cdot 10^{-138}$
5	$-4.72\ldots \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36\ldots \cdot 10^{-442}$	$5 - 3.14\ldots \cdot 10^{-105}$
6	$-2.97\ldots \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.78\ldots \cdot 10^{-547}$	$6 + 1.20\ldots \cdot 10^{-81}$
7	$+5.96\ldots \cdot 10^{-630}$	$\delta_{N,7} - \mu_{N,1} = -4.17\ldots \cdot 10^{-629}$	$7 + 1.08\ldots \cdot 10^{-61}$
8	$-9.25\ldots \cdot 10^{-692}$	$\delta_{N,8} - \mu_{N,1} = -2.00\ldots \cdot 10^{0}$	$-2.16\ldots \cdot 10^{691}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = -2.00\ldots \cdot 10^{0}$	$2 + 3.00\ldots \cdot 10^{-134}$
3	$-1.50\ldots \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50\ldots \cdot 10^{-134}$	$3 - 5.41\ldots \cdot 10^{-171}$
4	$-2.71\ldots \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08\ldots \cdot 10^{-304}$	$4 - 6.96\ldots \cdot 10^{-138}$
5	$-4.72\ldots \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36\ldots \cdot 10^{-442}$	$5 - 3.14\ldots \cdot 10^{-105}$
6	$-2.97\ldots \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.78\ldots \cdot 10^{-547}$	$6 + 1.20\ldots \cdot 10^{-81}$
7	$+5.96\ldots \cdot 10^{-630}$	$\delta_{N,7} - \mu_{N,1} = -4.17\ldots \cdot 10^{-629}$	$7 + 1.08\ldots \cdot 10^{-61}$
8	$-9.25\ldots \cdot 10^{-692}$	$\delta_{N,8} - \mu_{N,1} = -2.00\ldots \cdot 10^{0}$	$-2.16\ldots \cdot 10^{691}$

$$\frac{\mu_{N,2}}{\mu_{N,8}} = 1 + 5.42\ldots \cdot 10^{-305}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = -2.00\ldots \cdot 10^{0}$	$2 + 3.00\ldots \cdot 10^{-134}$
3	$-1.50\ldots \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50\ldots \cdot 10^{-134}$	$3 - 5.41\ldots \cdot 10^{-171}$
4	$-2.71\ldots \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08\ldots \cdot 10^{-304}$	$4 - 6.96\ldots \cdot 10^{-138}$
5	$-4.72\ldots \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36\ldots \cdot 10^{-442}$	$5 - 3.14\ldots \cdot 10^{-105}$
6	$-2.97\ldots \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.78\ldots \cdot 10^{-547}$	$6 + 1.20\ldots \cdot 10^{-81}$
7	$+5.96\ldots \cdot 10^{-630}$	$\delta_{N,7} - \mu_{N,1} = -4.17\ldots \cdot 10^{-629}$	$7 + 1.08\ldots \cdot 10^{-61}$
8	$-9.25\ldots \cdot 10^{-692}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} = +1.08\ldots \cdot 10^{-304}$	$+1.17\ldots \cdot 10^{387}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = -2.00\ldots \cdot 10^{0}$	$2 + 3.00\ldots \cdot 10^{-134}$
3	$-1.50\ldots \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50\ldots \cdot 10^{-134}$	$3 - 5.41\ldots \cdot 10^{-171}$
4	$-2.71\ldots \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08\ldots \cdot 10^{-304}$	$4 - 6.96\ldots \cdot 10^{-138}$
5	$-4.72\ldots \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36\ldots \cdot 10^{-442}$	$5 - 3.14\ldots \cdot 10^{-105}$
6	$-2.97\ldots \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.78\ldots \cdot 10^{-547}$	$6 + 1.20\ldots \cdot 10^{-81}$
7	$+5.96\ldots \cdot 10^{-630}$	$\delta_{N,7} - \mu_{N,1} = -4.17\ldots \cdot 10^{-629}$	$7 + 1.08\ldots \cdot 10^{-61}$
8	$-9.25\ldots \cdot 10^{-692}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} = +1.08\ldots \cdot 10^{-304}$	$+1.17\ldots \cdot 10^{387}$

$$\frac{\mu_{N,4}}{\mu_{N,8}} = 1 - 6.82\ldots \cdot 10^{-387}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = -2.00... \cdot 10^{0}$	$2 + 3.00... \cdot 10^{-134}$
3	$-1.50... \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50... \cdot 10^{-134}$	$3 - 5.41... \cdot 10^{-171}$
4	$-2.71... \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08... \cdot 10^{-304}$	$4 - 6.96... \cdot 10^{-138}$
5	$-4.72... \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36... \cdot 10^{-442}$	$5 - 3.14... \cdot 10^{-105}$
6	$-2.97... \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.78... \cdot 10^{-547}$	$6 + 1.20... \cdot 10^{-81}$
7	$+5.96... \cdot 10^{-630}$	$\delta_{N,7} - \mu_{N,1} = -4.17... \cdot 10^{-629}$	$7 + 1.08... \cdot 10^{-61}$
8	$-9.25... \cdot 10^{-692}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +7.40... \cdot 10^{-691}$	$8 - 2.52... \cdot 10^{-46}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = -2.00... \cdot 10^{0}$	$2 + 3.00... \cdot 10^{-134}$
3	$-1.50... \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50... \cdot 10^{-134}$	$3 - 5.41... \cdot 10^{-171}$
4	$-2.71... \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08... \cdot 10^{-304}$	$4 - 6.96... \cdot 10^{-138}$
5	$-4.72... \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36... \cdot 10^{-442}$	$5 - 3.14... \cdot 10^{-105}$
6	$-2.97... \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.78... \cdot 10^{-547}$	$6 + 1.20... \cdot 10^{-81}$
7	$+5.96... \cdot 10^{-630}$	$\delta_{N,7} - \mu_{N,1} = -4.17... \cdot 10^{-629}$	$7 + 1.08... \cdot 10^{-61}$
8	$-9.25... \cdot 10^{-692}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +7.40... \cdot 10^{-691}$	$8 - 2.52... \cdot 10^{-46}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
3	$-1.50\ldots \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50\ldots \cdot 10^{-134}$	$3 - 5.41\ldots \cdot 10^{-171}$
4	$-2.71\ldots \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08\ldots \cdot 10^{-304}$	$4 - 6.96\ldots \cdot 10^{-138}$
5	$-4.72\ldots \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36\ldots \cdot 10^{-442}$	$5 - 3.14\ldots \cdot 10^{-105}$
6	$-2.97\ldots \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.78\ldots \cdot 10^{-547}$	$6 + 1.20\ldots \cdot 10^{-81}$
7	$+5.96\ldots \cdot 10^{-630}$	$\delta_{N,7} - \mu_{N,1} = -4.17\ldots \cdot 10^{-629}$	$7 + 1.08\ldots \cdot 10^{-61}$
8	$-9.25\ldots \cdot 10^{-692}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +7.40\ldots \cdot 10^{-691}$	$8 - 2.52\ldots \cdot 10^{-46}$
9	$-2.91\ldots \cdot 10^{-738}$	$\delta_{N,9} - \mu_{N,1} = +4.50\ldots \cdot 10^{-134}$	$+1.54\ldots \cdot 10^{604}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
3	$-1.50... \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50... \cdot 10^{-134}$	$3 - 5.41... \cdot 10^{-171}$
4	$-2.71... \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08... \cdot 10^{-304}$	$4 - 6.96... \cdot 10^{-138}$
5	$-4.72... \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36... \cdot 10^{-442}$	$5 - 3.14... \cdot 10^{-105}$
6	$-2.97... \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.78... \cdot 10^{-547}$	$6 + 1.20... \cdot 10^{-81}$
7	$+5.96... \cdot 10^{-630}$	$\delta_{N,7} - \mu_{N,1} = -4.17... \cdot 10^{-629}$	$7 + 1.08... \cdot 10^{-61}$
8	$-9.25... \cdot 10^{-692}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +7.40... \cdot 10^{-691}$	$8 - 2.52... \cdot 10^{-46}$
9	$-2.91... \cdot 10^{-738}$	$\delta_{N,9} - \mu_{N,1} = +4.50... \cdot 10^{-134}$	$+1.54... \cdot 10^{604}$

$$\frac{\mu_{N,3}}{\mu_{N,9}} = 1 - 5.63... \cdot 10^{-604}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 3200$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
3	$-1.50\ldots \cdot 10^{-134}$	$\delta_{N,3} - \mu_{N,1} = +4.50\ldots \cdot 10^{-134}$	$3 - 5.41\ldots \cdot 10^{-171}$
4	$-2.71\ldots \cdot 10^{-305}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +1.08\ldots \cdot 10^{-304}$	$4 - 6.96\ldots \cdot 10^{-138}$
5	$-4.72\ldots \cdot 10^{-443}$	$\delta_{N,5} - \mu_{N,1} = +2.36\ldots \cdot 10^{-442}$	$5 - 3.14\ldots \cdot 10^{-105}$
6	$-2.97\ldots \cdot 10^{-548}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.78\ldots \cdot 10^{-547}$	$6 + 1.20\ldots \cdot 10^{-81}$
7	$+5.96\ldots \cdot 10^{-630}$	$\delta_{N,7} - \mu_{N,1} = -4.17\ldots \cdot 10^{-629}$	$7 + 1.08\ldots \cdot 10^{-61}$
8	$-9.25\ldots \cdot 10^{-692}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +7.40\ldots \cdot 10^{-691}$	$8 - 2.52\ldots \cdot 10^{-46}$
9	$-2.91\ldots \cdot 10^{-738}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +2.54\ldots \cdot 10^{-737}$	$+8.71\ldots \cdot 10^{0}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 4800$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
3	$-4.70... \cdot 10^{-201}$	$\delta_{N,3} - \mu_{N,1} = +1.41... \cdot 10^{-200}$	$3 + 1.63... \cdot 10^{-259}$
4	$+2.56... \cdot 10^{-460}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = -1.02... \cdot 10^{-459}$	$4 + 2.27... \cdot 10^{-211}$
5	$-1.45... \cdot 10^{-671}$	$\delta_{N,5} - \mu_{N,1} = +7.29... \cdot 10^{-671}$	$5 - 4.58... \cdot 10^{-165}$
6	$-1.33... \cdot 10^{-836}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +8.01... \cdot 10^{-836}$	$6 + 4.26... \cdot 10^{-129}$
7	$+9.49... \cdot 10^{-966}$	$\delta_{N,7} - \mu_{N,1} = -6.64... \cdot 10^{-965}$	$7 + 7.82... \cdot 10^{-102}$
8	$-1.06... \cdot 10^{-1067}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +8.49... \cdot 10^{-1067}$	$8 + 4.81... \cdot 10^{-81}$
9	$+6.38... \cdot 10^{-1149}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = -5.74... \cdot 10^{-1148}$	$9 - 2.66... \cdot 10^{-64}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 4800$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
3	$-4.70... \cdot 10^{-201}$	$\delta_{N,3} - \mu_{N,1} = +1.41... \cdot 10^{-200}$	$3 + 1.63... \cdot 10^{-259}$
4	$+2.56... \cdot 10^{-460}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = -1.02... \cdot 10^{-459}$	$4 + 2.27... \cdot 10^{-211}$
5	$-1.45... \cdot 10^{-671}$	$\delta_{N,5} - \mu_{N,1} = +7.29... \cdot 10^{-671}$	$5 - 4.58... \cdot 10^{-165}$
6	$-1.33... \cdot 10^{-836}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +8.01... \cdot 10^{-836}$	$6 + 4.26... \cdot 10^{-129}$
7	$+9.49... \cdot 10^{-966}$	$\delta_{N,7} - \mu_{N,1} = -6.64... \cdot 10^{-965}$	$7 + 7.82... \cdot 10^{-102}$
8	$-1.06... \cdot 10^{-1067}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +8.49... \cdot 10^{-1067}$	$8 + 4.81... \cdot 10^{-81}$
9	$+6.38... \cdot 10^{-1149}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = -5.74... \cdot 10^{-1148}$	$9 - 2.66... \cdot 10^{-64}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 4800$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
4	$+2.56\ldots \cdot 10^{-460}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = -1.02\ldots \cdot 10^{-459}$	$4 + 2.27\ldots \cdot 10^{-211}$
5	$-1.45\ldots \cdot 10^{-671}$	$\delta_{N,5} - \mu_{N,1} = +7.29\ldots \cdot 10^{-671}$	$5 - 4.58\ldots \cdot 10^{-165}$
6	$-1.33\ldots \cdot 10^{-836}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +8.01\ldots \cdot 10^{-836}$	$6 + 4.26\ldots \cdot 10^{-129}$
7	$+9.49\ldots \cdot 10^{-966}$	$\delta_{N,7} - \mu_{N,1} = -6.64\ldots \cdot 10^{-965}$	$7 + 7.82\ldots \cdot 10^{-102}$
8	$-1.06\ldots \cdot 10^{-1067}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +8.49\ldots \cdot 10^{-1067}$	$8 + 4.81\ldots \cdot 10^{-81}$
9	$+6.38\ldots \cdot 10^{-1149}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = -5.74\ldots \cdot 10^{-1148}$	$9 - 2.66\ldots \cdot 10^{-64}$
10	$+1.88\ldots \cdot 10^{-1213}$	$\delta_{N,10} - \mu_{N,1} = -2.00\ldots \cdot 10^{0}$	$+1.05\ldots \cdot 10^{1213}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\qquad$ $\mu_{N,1} = \delta_{N,1} = 1$ $\qquad$ $N = 4800$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
4	$+2.56\ldots \cdot 10^{-460}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = -1.02\ldots \cdot 10^{-459}$	$4 + 2.27\ldots \cdot 10^{-211}$
5	$-1.45\ldots \cdot 10^{-671}$	$\delta_{N,5} - \mu_{N,1} = +7.29\ldots \cdot 10^{-671}$	$5 - 4.58\ldots \cdot 10^{-165}$
6	$-1.33\ldots \cdot 10^{-836}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +8.01\ldots \cdot 10^{-836}$	$6 + 4.26\ldots \cdot 10^{-129}$
7	$+9.49\ldots \cdot 10^{-966}$	$\delta_{N,7} - \mu_{N,1} = -6.64\ldots \cdot 10^{-965}$	$7 + 7.82\ldots \cdot 10^{-102}$
8	$-1.06\ldots \cdot 10^{-1067}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +8.49\ldots \cdot 10^{-1067}$	$8 + 4.81\ldots \cdot 10^{-81}$
9	$+6.38\ldots \cdot 10^{-1149}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = -5.74\ldots \cdot 10^{-1148}$	$9 - 2.66\ldots \cdot 10^{-64}$
10	$+1.88\ldots \cdot 10^{-1213}$	$\delta_{N,10} - \mu_{N,1} = -2.00\ldots \cdot 10^{0}$	$+1.05\ldots \cdot 10^{1213}$

$$\frac{\mu_{N,2}}{\mu_{N,10}} = 1 + 3.64\ldots \cdot 10^{-671}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 4800$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
4	$+2.56\ldots \cdot 10^{-460}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = -1.02\ldots \cdot 10^{-459}$	$4 + 2.27\ldots \cdot 10^{-211}$
5	$-1.45\ldots \cdot 10^{-671}$	$\delta_{N,5} - \mu_{N,1} = +7.29\ldots \cdot 10^{-671}$	$5 - 4.58\ldots \cdot 10^{-165}$
6	$-1.33\ldots \cdot 10^{-836}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +8.01\ldots \cdot 10^{-836}$	$6 + 4.26\ldots \cdot 10^{-129}$
7	$+9.49\ldots \cdot 10^{-966}$	$\delta_{N,7} - \mu_{N,1} = -6.64\ldots \cdot 10^{-965}$	$7 + 7.82\ldots \cdot 10^{-102}$
8	$-1.06\ldots \cdot 10^{-1067}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +8.49\ldots \cdot 10^{-1067}$	$8 + 4.81\ldots \cdot 10^{-81}$
9	$+6.38\ldots \cdot 10^{-1149}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = -5.74\ldots \cdot 10^{-1148}$	$9 - 2.66\ldots \cdot 10^{-64}$
10	$+1.88\ldots \cdot 10^{-1213}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} = +7.29\ldots \cdot 10^{-671}$	$-3.86\ldots \cdot 10^{542}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 4800$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
4	$+2.56\ldots \cdot 10^{-460}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = -1.02\ldots \cdot 10^{-459}$	$4 + 2.27\ldots \cdot 10^{-211}$
5	$-1.45\ldots \cdot 10^{-671}$	$\delta_{N,5} - \mu_{N,1} = +7.29\ldots \cdot 10^{-671}$	$5 - 4.58\ldots \cdot 10^{-165}$
6	$-1.33\ldots \cdot 10^{-836}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +8.01\ldots \cdot 10^{-836}$	$6 + 4.26\ldots \cdot 10^{-129}$
7	$+9.49\ldots \cdot 10^{-966}$	$\delta_{N,7} - \mu_{N,1} = -6.64\ldots \cdot 10^{-965}$	$7 + 7.82\ldots \cdot 10^{-102}$
8	$-1.06\ldots \cdot 10^{-1067}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +8.49\ldots \cdot 10^{-1067}$	$8 + 4.81\ldots \cdot 10^{-81}$
9	$+6.38\ldots \cdot 10^{-1149}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = -5.74\ldots \cdot 10^{-1148}$	$9 - 2.66\ldots \cdot 10^{-64}$
10	$+1.88\ldots \cdot 10^{-1213}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} = +7.29\ldots \cdot 10^{-671}$	$-3.86\ldots \cdot 10^{542}$

$$\frac{\mu_{N,5}}{\mu_{N,10}} = 1 + 2.58\ldots \cdot 10^{-542}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 4800$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
4	$+2.56... \cdot 10^{-460}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = -1.02... \cdot 10^{-459}$	$4 + 2.27... \cdot 10^{-211}$
5	$-1.45... \cdot 10^{-671}$	$\delta_{N,5} - \mu_{N,1} = +7.29... \cdot 10^{-671}$	$5 - 4.58... \cdot 10^{-165}$
6	$-1.33... \cdot 10^{-836}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +8.01... \cdot 10^{-836}$	$6 + 4.26... \cdot 10^{-129}$
7	$+9.49... \cdot 10^{-966}$	$\delta_{N,7} - \mu_{N,1} = -6.64... \cdot 10^{-965}$	$7 + 7.82... \cdot 10^{-102}$
8	$-1.06... \cdot 10^{-1067}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +8.49... \cdot 10^{-1067}$	$8 + 4.81... \cdot 10^{-81}$
9	$+6.38... \cdot 10^{-1149}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = -5.74... \cdot 10^{-1148}$	$9 - 2.66... \cdot 10^{-64}$
10	$+1.88... \cdot 10^{-1213}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = -1.88... \cdot 10^{-1212}$	$10 + 2.81... \cdot 10^{-35}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 4800$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
4	$+2.56... \cdot 10^{-460}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = -1.02... \cdot 10^{-459}$	$4 + 2.27... \cdot 10^{-211}$
5	$-1.45... \cdot 10^{-671}$	$\delta_{N,5} - \mu_{N,1} = +7.29... \cdot 10^{-671}$	$5 - 4.58... \cdot 10^{-165}$
6	$-1.33... \cdot 10^{-836}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +8.01... \cdot 10^{-836}$	$6 + 4.26... \cdot 10^{-129}$
7	$+9.49... \cdot 10^{-966}$	$\delta_{N,7} - \mu_{N,1} = -6.64... \cdot 10^{-965}$	$7 + 7.82... \cdot 10^{-102}$
8	$-1.06... \cdot 10^{-1067}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +8.49... \cdot 10^{-1067}$	$8 + 4.81... \cdot 10^{-81}$
9	$+6.38... \cdot 10^{-1149}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = -5.74... \cdot 10^{-1148}$	$9 - 2.66... \cdot 10^{-64}$
10	$+1.88... \cdot 10^{-1213}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = -1.88... \cdot 10^{-1212}$	$10 + 2.81... \cdot 10^{-35}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 4800$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
5	$-1.45\ldots \cdot 10^{-671}$	$\delta_{N,5} - \mu_{N,1} = +7.29\ldots \cdot 10^{-671}$	$5 - 4.58\ldots \cdot 10^{-165}$
6	$-1.33\ldots \cdot 10^{-836}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +8.01\ldots \cdot 10^{-836}$	$6 + 4.26\ldots \cdot 10^{-129}$
7	$+9.49\ldots \cdot 10^{-966}$	$\delta_{N,7} - \mu_{N,1} = -6.64\ldots \cdot 10^{-965}$	$7 + 7.82\ldots \cdot 10^{-102}$
8	$-1.06\ldots \cdot 10^{-1067}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +8.49\ldots \cdot 10^{-1067}$	$8 + 4.81\ldots \cdot 10^{-81}$
9	$+6.38\ldots \cdot 10^{-1149}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = -5.74\ldots \cdot 10^{-1148}$	$9 - 2.66\ldots \cdot 10^{-64}$
10	$+1.88\ldots \cdot 10^{-1213}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = -1.88\ldots \cdot 10^{-1212}$	$10 + 2.81\ldots \cdot 10^{-35}$
11	$-5.32\ldots \cdot 10^{-1249}$	$\delta_{N,11} - \mu_{N,1} = +2.13\ldots \cdot 10^{-1249}$	$+4.00\ldots \cdot 10^{-1}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 8000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
5	$-9.62... \cdot 10^{-517}$	$\delta_{N,5} - \mu_{N,1} = +4.81... \cdot 10^{-516}$	$5 - 1.07... \cdot 10^{-287}$
6	$-2.06... \cdot 10^{-804}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.24... \cdot 10^{-803}$	$6 - 2.99... \cdot 10^{-229}$
7	$-1.03... \cdot 10^{-1033}$	$\delta_{N,7} - \mu_{N,1} = +7.22... \cdot 10^{-1033}$	$7 - 2.34... \cdot 10^{-184}$
8	$-3.45... \cdot 10^{-1218}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +2.76... \cdot 10^{-1217}$	$8 - 2.11... \cdot 10^{-148}$
9	$-9.12... \cdot 10^{-1367}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +8.21... \cdot 10^{-1366}$	$9 - 1.98... \cdot 10^{-121}$
10	$-2.01... \cdot 10^{-1488}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +2.01... \cdot 10^{-1487}$	$10 - 1.67... \cdot 10^{-100}$
11	$-3.37... \cdot 10^{-1589}$	$\delta_{N,11} - \mu_{N,1} = +3.70... \cdot 10^{-1588}$	$11 - 1.01... \cdot 10^{-80}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 8000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
5	$-9.62... \cdot 10^{-517}$	$\delta_{N,5} - \mu_{N,1} = +4.81... \cdot 10^{-516}$	$5 - 1.07... \cdot 10^{-287}$
6	$-2.06... \cdot 10^{-804}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.24... \cdot 10^{-803}$	$6 - 2.99... \cdot 10^{-229}$
7	$-1.03... \cdot 10^{-1033}$	$\delta_{N,7} - \mu_{N,1} = +7.22... \cdot 10^{-1033}$	$7 - 2.34... \cdot 10^{-184}$
8	$-3.45... \cdot 10^{-1218}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +2.76... \cdot 10^{-1217}$	$8 - 2.11... \cdot 10^{-148}$
9	$-9.12... \cdot 10^{-1367}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +8.21... \cdot 10^{-1366}$	$9 - 1.98... \cdot 10^{-121}$
10	$-2.01... \cdot 10^{-1488}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +2.01... \cdot 10^{-1487}$	$10 - 1.67... \cdot 10^{-100}$
11	$-3.37... \cdot 10^{-1589}$	$\delta_{N,11} - \mu_{N,1} = +3.70... \cdot 10^{-1588}$	$11 - 1.01... \cdot 10^{-80}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 8000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
6	$-2.06\ldots \cdot 10^{-804}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.24\ldots \cdot 10^{-803}$	$6 - 2.99\ldots \cdot 10^{-229}$
7	$-1.03\ldots \cdot 10^{-1033}$	$\delta_{N,7} - \mu_{N,1} = +7.22\ldots \cdot 10^{-1033}$	$7 - 2.34\ldots \cdot 10^{-184}$
8	$-3.45\ldots \cdot 10^{-1218}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +2.76\ldots \cdot 10^{-1217}$	$8 - 2.11\ldots \cdot 10^{-148}$
9	$-9.12\ldots \cdot 10^{-1367}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +8.21\ldots \cdot 10^{-1366}$	$9 - 1.98\ldots \cdot 10^{-121}$
10	$-2.01\ldots \cdot 10^{-1488}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +2.01\ldots \cdot 10^{-1487}$	$10 - 1.67\ldots \cdot 10^{-100}$
11	$-3.37\ldots \cdot 10^{-1589}$	$\delta_{N,11} - \mu_{N,1} = +3.70\ldots \cdot 10^{-1588}$	$11 - 1.01\ldots \cdot 10^{-80}$
12	$-3.09\ldots \cdot 10^{-1670}$	$\delta_{N,12} - \mu_{N,1} = +1.98\ldots \cdot 10^{285}$	$+6.41\ldots \cdot 10^{1954}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 8000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
6	$-2.06\ldots \cdot 10^{-804}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.24\ldots \cdot 10^{-803}$	$6 - 2.99\ldots \cdot 10^{-229}$
7	$-1.03\ldots \cdot 10^{-1033}$	$\delta_{N,7} - \mu_{N,1} = +7.22\ldots \cdot 10^{-1033}$	$7 - 2.34\ldots \cdot 10^{-184}$
8	$-3.45\ldots \cdot 10^{-1218}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +2.76\ldots \cdot 10^{-1217}$	$8 - 2.11\ldots \cdot 10^{-148}$
9	$-9.12\ldots \cdot 10^{-1367}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +8.21\ldots \cdot 10^{-1366}$	$9 - 1.98\ldots \cdot 10^{-121}$
10	$-2.01\ldots \cdot 10^{-1488}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +2.01\ldots \cdot 10^{-1487}$	$10 - 1.67\ldots \cdot 10^{-100}$
11	$-3.37\ldots \cdot 10^{-1589}$	$\delta_{N,11} - \mu_{N,1} = +3.70\ldots \cdot 10^{-1588}$	$11 - 1.01\ldots \cdot 10^{-80}$
12	$-3.09\ldots \cdot 10^{-1670}$	$\delta_{N,12} - \mu_{N,1} = +1.98\ldots \cdot 10^{285}$	$+6.41\ldots \cdot 10^{1954}$

$$\frac{\mu_{N,2}}{\mu_{N,12}} = 1 - 3.00\ldots \cdot 10^{0}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 8000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
6	$-2.06\ldots \cdot 10^{-804}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.24\ldots \cdot 10^{-803}$	$6 - 2.99\ldots \cdot 10^{-229}$
7	$-1.03\ldots \cdot 10^{-1033}$	$\delta_{N,7} - \mu_{N,1} = +7.22\ldots \cdot 10^{-1033}$	$7 - 2.34\ldots \cdot 10^{-184}$
8	$-3.45\ldots \cdot 10^{-1218}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +2.76\ldots \cdot 10^{-1217}$	$8 - 2.11\ldots \cdot 10^{-148}$
9	$-9.12\ldots \cdot 10^{-1367}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +8.21\ldots \cdot 10^{-1366}$	$9 - 1.98\ldots \cdot 10^{-121}$
10	$-2.01\ldots \cdot 10^{-1488}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +2.01\ldots \cdot 10^{-1487}$	$10 - 1.67\ldots \cdot 10^{-100}$
11	$-3.37\ldots \cdot 10^{-1589}$	$\delta_{N,11} - \mu_{N,1} = +3.70\ldots \cdot 10^{-1588}$	$11 - 1.01\ldots \cdot 10^{-80}$
12	$-3.09\ldots \cdot 10^{-1670}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} = +5.96\ldots \cdot 10^{285}$	$+1.92\ldots \cdot 10^{1955}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 8000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
6	$-2.06\ldots \cdot 10^{-804}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.24\ldots \cdot 10^{-803}$	$6 - 2.99\ldots \cdot 10^{-229}$
7	$-1.03\ldots \cdot 10^{-1033}$	$\delta_{N,7} - \mu_{N,1} = +7.22\ldots \cdot 10^{-1033}$	$7 - 2.34\ldots \cdot 10^{-184}$
8	$-3.45\ldots \cdot 10^{-1218}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +2.76\ldots \cdot 10^{-1217}$	$8 - 2.11\ldots \cdot 10^{-148}$
9	$-9.12\ldots \cdot 10^{-1367}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +8.21\ldots \cdot 10^{-1366}$	$9 - 1.98\ldots \cdot 10^{-121}$
10	$-2.01\ldots \cdot 10^{-1488}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +2.01\ldots \cdot 10^{-1487}$	$10 - 1.67\ldots \cdot 10^{-100}$
11	$-3.37\ldots \cdot 10^{-1589}$	$\delta_{N,11} - \mu_{N,1} = +3.70\ldots \cdot 10^{-1588}$	$11 - 1.01\ldots \cdot 10^{-80}$
12	$-3.09\ldots \cdot 10^{-1670}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} = +5.96\ldots \cdot 10^{285}$	$+1.92\ldots \cdot 10^{1955}$

$$\frac{\mu_{N,3}}{\mu_{N,12}} = 1 - 2.61\ldots \cdot 10^{-439}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 8000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
6	$-2.06\ldots \cdot 10^{-804}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.24\ldots \cdot 10^{-803}$	$6 - 2.99\ldots \cdot 10^{-229}$
7	$-1.03\ldots \cdot 10^{-1033}$	$\delta_{N,7} - \mu_{N,1} = +7.22\ldots \cdot 10^{-1033}$	$7 - 2.34\ldots \cdot 10^{-184}$
8	$-3.45\ldots \cdot 10^{-1218}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +2.76\ldots \cdot 10^{-1217}$	$8 - 2.11\ldots \cdot 10^{-148}$
9	$-9.12\ldots \cdot 10^{-1367}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +8.21\ldots \cdot 10^{-1366}$	$9 - 1.98\ldots \cdot 10^{-121}$
10	$-2.01\ldots \cdot 10^{-1488}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +2.01\ldots \cdot 10^{-1487}$	$10 - 1.67\ldots \cdot 10^{-100}$
11	$-3.37\ldots \cdot 10^{-1589}$	$\delta_{N,11} - \mu_{N,1} = +3.70\ldots \cdot 10^{-1588}$	$11 - 1.01\ldots \cdot 10^{-80}$
12	$-3.09\ldots \cdot 10^{-1670}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.56\ldots \cdot 10^{-153}$	$+5.03\ldots \cdot 10^{1516}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 8000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
6	$-2.06\ldots \cdot 10^{-804}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.24\ldots \cdot 10^{-803}$	$6 - 2.99\ldots \cdot 10^{-229}$
7	$-1.03\ldots \cdot 10^{-1033}$	$\delta_{N,7} - \mu_{N,1} = +7.22\ldots \cdot 10^{-1033}$	$7 - 2.34\ldots \cdot 10^{-184}$
8	$-3.45\ldots \cdot 10^{-1218}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +2.76\ldots \cdot 10^{-1217}$	$8 - 2.11\ldots \cdot 10^{-148}$
9	$-9.12\ldots \cdot 10^{-1367}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +8.21\ldots \cdot 10^{-1366}$	$9 - 1.98\ldots \cdot 10^{-121}$
10	$-2.01\ldots \cdot 10^{-1488}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +2.01\ldots \cdot 10^{-1487}$	$10 - 1.67\ldots \cdot 10^{-100}$
11	$-3.37\ldots \cdot 10^{-1589}$	$\delta_{N,11} - \mu_{N,1} = +3.70\ldots \cdot 10^{-1588}$	$11 - 1.01\ldots \cdot 10^{-80}$
12	$-3.09\ldots \cdot 10^{-1670}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.56\ldots \cdot 10^{-153}$	$+5.03\ldots \cdot 10^{1516}$

$$\frac{\mu_{N,4}}{\mu_{N,12}} = 1 - 7.94\ldots \cdot 10^{-651}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 8000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
6	$-2.06\ldots \cdot 10^{-804}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.24\ldots \cdot 10^{-803}$	$6 - 2.99\ldots \cdot 10^{-229}$
7	$-1.03\ldots \cdot 10^{-1033}$	$\delta_{N,7} - \mu_{N,1} = +7.22\ldots \cdot 10^{-1033}$	$7 - 2.34\ldots \cdot 10^{-184}$
8	$-3.45\ldots \cdot 10^{-1218}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +2.76\ldots \cdot 10^{-1217}$	$8 - 2.11\ldots \cdot 10^{-148}$
9	$-9.12\ldots \cdot 10^{-1367}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +8.21\ldots \cdot 10^{-1366}$	$9 - 1.98\ldots \cdot 10^{-121}$
10	$-2.01\ldots \cdot 10^{-1488}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +2.01\ldots \cdot 10^{-1487}$	$10 - 1.67\ldots \cdot 10^{-100}$
11	$-3.37\ldots \cdot 10^{-1589}$	$\delta_{N,11} - \mu_{N,1} = +3.70\ldots \cdot 10^{-1588}$	$11 - 1.01\ldots \cdot 10^{-80}$
12	$-3.09\ldots \cdot 10^{-1670}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} = +1.24\ldots \cdot 10^{-803}$	$+4.00\ldots \cdot 10^{866}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 8000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
6	$-2.06\ldots \cdot 10^{-804}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.24\ldots \cdot 10^{-803}$	$6 - 2.99\ldots \cdot 10^{-229}$
7	$-1.03\ldots \cdot 10^{-1033}$	$\delta_{N,7} - \mu_{N,1} = +7.22\ldots \cdot 10^{-1033}$	$7 - 2.34\ldots \cdot 10^{-184}$
8	$-3.45\ldots \cdot 10^{-1218}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +2.76\ldots \cdot 10^{-1217}$	$8 - 2.11\ldots \cdot 10^{-148}$
9	$-9.12\ldots \cdot 10^{-1367}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +8.21\ldots \cdot 10^{-1366}$	$9 - 1.98\ldots \cdot 10^{-121}$
10	$-2.01\ldots \cdot 10^{-1488}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +2.01\ldots \cdot 10^{-1487}$	$10 - 1.67\ldots \cdot 10^{-100}$
11	$-3.37\ldots \cdot 10^{-1589}$	$\delta_{N,11} - \mu_{N,1} = +3.70\ldots \cdot 10^{-1588}$	$11 - 1.01\ldots \cdot 10^{-80}$
12	$-3.09\ldots \cdot 10^{-1670}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} = +1.24\ldots \cdot 10^{-803}$	$+4.00\ldots \cdot 10^{866}$

$$\frac{\mu_{N,6}}{\mu_{N,12}} = 1 - 2.99\ldots \cdot 10^{-866}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 8000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
6	$-2.06... \cdot 10^{-804}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.24... \cdot 10^{-803}$	$6 - 2.99... \cdot 10^{-229}$
7	$-1.03... \cdot 10^{-1033}$	$\delta_{N,7} - \mu_{N,1} = +7.22... \cdot 10^{-1033}$	$7 - 2.34... \cdot 10^{-184}$
8	$-3.45... \cdot 10^{-1218}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +2.76... \cdot 10^{-1217}$	$8 - 2.11... \cdot 10^{-148}$
9	$-9.12... \cdot 10^{-1367}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +8.21... \cdot 10^{-1366}$	$9 - 1.98... \cdot 10^{-121}$
10	$-2.01... \cdot 10^{-1488}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +2.01... \cdot 10^{-1487}$	$10 - 1.67... \cdot 10^{-100}$
11	$-3.37... \cdot 10^{-1589}$	$\delta_{N,11} - \mu_{N,1} = +3.70... \cdot 10^{-1588}$	$11 - 1.01... \cdot 10^{-80}$
12	$-3.09... \cdot 10^{-1670}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +3.71... \cdot 10^{-1669}$	$12 + 2.07... \cdot 10^{-67}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\qquad \mu_{N,1} = \delta_{N,1} = 1$ $\qquad N = 8000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
6	$-2.06... \cdot 10^{-804}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +1.24... \cdot 10^{-803}$	$6 - 2.99... \cdot 10^{-229}$
7	$-1.03... \cdot 10^{-1033}$	$\delta_{N,7} - \mu_{N,1} = +7.22... \cdot 10^{-1033}$	$7 - 2.34... \cdot 10^{-184}$
8	$-3.45... \cdot 10^{-1218}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +2.76... \cdot 10^{-1217}$	$8 - 2.11... \cdot 10^{-148}$
9	$-9.12... \cdot 10^{-1367}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +8.21... \cdot 10^{-1366}$	$9 - 1.98... \cdot 10^{-121}$
10	$-2.01... \cdot 10^{-1488}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +2.01... \cdot 10^{-1487}$	$10 - 1.67... \cdot 10^{-100}$
11	$-3.37... \cdot 10^{-1589}$	$\delta_{N,11} - \mu_{N,1} = +3.70... \cdot 10^{-1588}$	$11 - 1.01... \cdot 10^{-80}$
12	$-3.09... \cdot 10^{-1670}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +3.71... \cdot 10^{-1669}$	$12 + 2.07... \cdot 10^{-67}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 8000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
7	$-1.03... \cdot 10^{-1033}$	$\delta_{N,7} - \mu_{N,1} = +7.22... \cdot 10^{-1033}$	$7 - 2.34... \cdot 10^{-184}$
8	$-3.45... \cdot 10^{-1218}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +2.76... \cdot 10^{-1217}$	$8 - 2.11... \cdot 10^{-148}$
9	$-9.12... \cdot 10^{-1367}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +8.21... \cdot 10^{-1366}$	$9 - 1.98... \cdot 10^{-121}$
10	$-2.01... \cdot 10^{-1488}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +2.01... \cdot 10^{-1487}$	$10 - 1.67... \cdot 10^{-100}$
11	$-3.37... \cdot 10^{-1589}$	$\delta_{N,11} - \mu_{N,1} = +3.70... \cdot 10^{-1588}$	$11 - 1.01... \cdot 10^{-80}$
12	$-3.09... \cdot 10^{-1670}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +3.71... \cdot 10^{-1669}$	$12 + 2.07... \cdot 10^{-67}$
13	$+5.36... \cdot 10^{-1738}$	$\delta_{N,13} - \mu_{N,1} = -6.97... \cdot 10^{-1737}$	$13 - 2.13... \cdot 10^{-4}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 9600$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
7	$-2.24... \cdot 10^{-779}$	$\delta_{N,7} - \mu_{N,1} = +1.57... \cdot 10^{-778}$	$7 - 5.54... \cdot 10^{-224}$
8	$-1.78... \cdot 10^{-1003}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +1.42... \cdot 10^{-1002}$	$8 - 1.41... \cdot 10^{-183}$
9	$-3.14... \cdot 10^{-1187}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +2.83... \cdot 10^{-1186}$	$9 - 2.59... \cdot 10^{-151}$
10	$-9.07... \cdot 10^{-1339}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +9.07... \cdot 10^{-1338}$	$10 + 1.84... \cdot 10^{-125}$
11	$+1.66... \cdot 10^{-1464}$	$\delta_{N,11} - \mu_{N,1} = -1.83... \cdot 10^{-1463}$	$11 + 2.94... \cdot 10^{-103}$
12	$-4.46... \cdot 10^{-1568}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +5.35... \cdot 10^{-1567}$	$12 + 3.63... \cdot 10^{-86}$
13	$+1.35... \cdot 10^{-1654}$	$\delta_{N,13} - \mu_{N,1} = -1.75... \cdot 10^{-1653}$	$13 - 3.19... \cdot 10^{-70}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 9600$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
7	$-2.24... \cdot 10^{-779}$	$\delta_{N,7} - \mu_{N,1} = +1.57... \cdot 10^{-778}$	$7 - 5.54... \cdot 10^{-224}$
8	$-1.78... \cdot 10^{-1003}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +1.42... \cdot 10^{-1002}$	$8 - 1.41... \cdot 10^{-183}$
9	$-3.14... \cdot 10^{-1187}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +2.83... \cdot 10^{-1186}$	$9 - 2.59... \cdot 10^{-151}$
10	$-9.07... \cdot 10^{-1339}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +9.07... \cdot 10^{-1338}$	$10 + 1.84... \cdot 10^{-125}$
11	$+1.66... \cdot 10^{-1464}$	$\delta_{N,11} - \mu_{N,1} = -1.83... \cdot 10^{-1463}$	$11 + 2.94... \cdot 10^{-103}$
12	$-4.46... \cdot 10^{-1568}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +5.35... \cdot 10^{-1567}$	$12 + 3.63... \cdot 10^{-86}$
13	$+1.35... \cdot 10^{-1654}$	$\delta_{N,13} - \mu_{N,1} = -1.75... \cdot 10^{-1653}$	$13 - 3.19... \cdot 10^{-70}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 9600$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
8	$-1.78\ldots \cdot 10^{-1003}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +1.42\ldots \cdot 10^{-1002}$	$8 - 1.41\ldots \cdot 10^{-183}$
9	$-3.14\ldots \cdot 10^{-1187}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +2.83\ldots \cdot 10^{-1186}$	$9 - 2.59\ldots \cdot 10^{-151}$
10	$-9.07\ldots \cdot 10^{-1339}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +9.07\ldots \cdot 10^{-1338}$	$10 + 1.84\ldots \cdot 10^{-125}$
11	$+1.66\ldots \cdot 10^{-1464}$	$\delta_{N,11} - \mu_{N,1} = -1.83\ldots \cdot 10^{-1463}$	$11 + 2.94\ldots \cdot 10^{-103}$
12	$-4.46\ldots \cdot 10^{-1568}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +5.35\ldots \cdot 10^{-1567}$	$12 + 3.63\ldots \cdot 10^{-86}$
13	$+1.35\ldots \cdot 10^{-1654}$	$\delta_{N,13} - \mu_{N,1} = -1.75\ldots \cdot 10^{-1653}$	$13 - 3.19\ldots \cdot 10^{-70}$
14	$+3.31\ldots \cdot 10^{-1725}$	$\delta_{N,14} - \mu_{N,1} = -3.52\ldots \cdot 10^{818}$	$+1.06\ldots \cdot 10^{2543}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 9600$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
8	$-1.78\ldots \cdot 10^{-1003}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +1.42\ldots \cdot 10^{-1002}$	$8 - 1.41\ldots \cdot 10^{-183}$
9	$-3.14\ldots \cdot 10^{-1187}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +2.83\ldots \cdot 10^{-1186}$	$9 - 2.59\ldots \cdot 10^{-151}$
10	$-9.07\ldots \cdot 10^{-1339}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +9.07\ldots \cdot 10^{-1338}$	$10 + 1.84\ldots \cdot 10^{-125}$
11	$+1.66\ldots \cdot 10^{-1464}$	$\delta_{N,11} - \mu_{N,1} = -1.83\ldots \cdot 10^{-1463}$	$11 + 2.94\ldots \cdot 10^{-103}$
12	$-4.46\ldots \cdot 10^{-1568}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +5.35\ldots \cdot 10^{-1567}$	$12 + 3.63\ldots \cdot 10^{-86}$
13	$+1.35\ldots \cdot 10^{-1654}$	$\delta_{N,13} - \mu_{N,1} = -1.75\ldots \cdot 10^{-1653}$	$13 - 3.19\ldots \cdot 10^{-70}$
14	$+3.31\ldots \cdot 10^{-1725}$	$\delta_{N,14} - \mu_{N,1} = -3.52\ldots \cdot 10^{818}$	$+1.06\ldots \cdot 10^{2543}$

$$\frac{\mu_{N,2}}{\mu_{N,14}} = 1 + 4.45\ldots \cdot 10^{-1597}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 9600$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
8	$-1.78\ldots \cdot 10^{-1003}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +1.42\ldots \cdot 10^{-1002}$	$8 - 1.41\ldots \cdot 10^{-183}$
9	$-3.14\ldots \cdot 10^{-1187}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +2.83\ldots \cdot 10^{-1186}$	$9 - 2.59\ldots \cdot 10^{-151}$
10	$-9.07\ldots \cdot 10^{-1339}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +9.07\ldots \cdot 10^{-1338}$	$10 + 1.84\ldots \cdot 10^{-125}$
11	$+1.66\ldots \cdot 10^{-1464}$	$\delta_{N,11} - \mu_{N,1} = -1.83\ldots \cdot 10^{-1463}$	$11 + 2.94\ldots \cdot 10^{-103}$
12	$-4.46\ldots \cdot 10^{-1568}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +5.35\ldots \cdot 10^{-1567}$	$12 + 3.63\ldots \cdot 10^{-86}$
13	$+1.35\ldots \cdot 10^{-1654}$	$\delta_{N,13} - \mu_{N,1} = -1.75\ldots \cdot 10^{-1653}$	$13 - 3.19\ldots \cdot 10^{-70}$
14	$+3.31\ldots \cdot 10^{-1725}$	$\delta_{N,14} - \mu_{N,1} - \mu_{N,2} = +1.57\ldots \cdot 10^{-778}$	$-4.74\ldots \cdot 10^{946}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 9600$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
8	$-1.78\ldots \cdot 10^{-1003}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +1.42\ldots \cdot 10^{-1002}$	$8 - 1.41\ldots \cdot 10^{-183}$
9	$-3.14\ldots \cdot 10^{-1187}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +2.83\ldots \cdot 10^{-1186}$	$9 - 2.59\ldots \cdot 10^{-151}$
10	$-9.07\ldots \cdot 10^{-1339}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +9.07\ldots \cdot 10^{-1338}$	$10 + 1.84\ldots \cdot 10^{-125}$
11	$+1.66\ldots \cdot 10^{-1464}$	$\delta_{N,11} - \mu_{N,1} = -1.83\ldots \cdot 10^{-1463}$	$11 + 2.94\ldots \cdot 10^{-103}$
12	$-4.46\ldots \cdot 10^{-1568}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +5.35\ldots \cdot 10^{-1567}$	$12 + 3.63\ldots \cdot 10^{-86}$
13	$+1.35\ldots \cdot 10^{-1654}$	$\delta_{N,13} - \mu_{N,1} = -1.75\ldots \cdot 10^{-1653}$	$13 - 3.19\ldots \cdot 10^{-70}$
14	$+3.31\ldots \cdot 10^{-1725}$	$\delta_{N,14} - \mu_{N,1} - \mu_{N,2} = +1.57\ldots \cdot 10^{-778}$	$-4.74\ldots \cdot 10^{946}$

$$\frac{\mu_{N,7}}{\mu_{N,14}} = 1 + 2.95\ldots \cdot 10^{-946}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 9600$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
8	$-1.78\ldots \cdot 10^{-1003}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +1.42\ldots \cdot 10^{-1002}$	$8 - 1.41\ldots \cdot 10^{-183}$
9	$-3.14\ldots \cdot 10^{-1187}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +2.83\ldots \cdot 10^{-1186}$	$9 - 2.59\ldots \cdot 10^{-151}$
10	$-9.07\ldots \cdot 10^{-1339}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +9.07\ldots \cdot 10^{-1338}$	$10 + 1.84\ldots \cdot 10^{-125}$
11	$+1.66\ldots \cdot 10^{-1464}$	$\delta_{N,11} - \mu_{N,1} = -1.83\ldots \cdot 10^{-1463}$	$11 + 2.94\ldots \cdot 10^{-103}$
12	$-4.46\ldots \cdot 10^{-1568}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +5.35\ldots \cdot 10^{-1567}$	$12 + 3.63\ldots \cdot 10^{-86}$
13	$+1.35\ldots \cdot 10^{-1654}$	$\delta_{N,13} - \mu_{N,1} = -1.75\ldots \cdot 10^{-1653}$	$13 - 3.19\ldots \cdot 10^{-70}$
14	$+3.31\ldots \cdot 10^{-1725}$	$\delta_{N,14} - \mu_{N,1} - \mu_{N,2} - \mu_{N,7} = -4.64\ldots \cdot 10^{-1724}$	$14 - 1.59\ldots \cdot 10^{-6}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 9600$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
8	$-1.78... \cdot 10^{-1003}$	$\delta_{N,8} - \mu_{N,1} - \mu_{N,2} - \mu_{N,4} = +1.42... \cdot 10^{-1002}$	$8 - 1.41... \cdot 10^{-183}$
9	$-3.14... \cdot 10^{-1187}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +2.83... \cdot 10^{-1186}$	$9 - 2.59... \cdot 10^{-151}$
10	$-9.07... \cdot 10^{-1339}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +9.07... \cdot 10^{-1338}$	$10 + 1.84... \cdot 10^{-125}$
11	$+1.66... \cdot 10^{-1464}$	$\delta_{N,11} - \mu_{N,1} = -1.83... \cdot 10^{-1463}$	$11 + 2.94... \cdot 10^{-103}$
12	$-4.46... \cdot 10^{-1568}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +5.35... \cdot 10^{-1567}$	$12 + 3.63... \cdot 10^{-86}$
13	$+1.35... \cdot 10^{-1654}$	$\delta_{N,13} - \mu_{N,1} = -1.75... \cdot 10^{-1653}$	$13 - 3.19... \cdot 10^{-70}$
14	$+3.31... \cdot 10^{-1725}$	$\delta_{N,14} - \mu_{N,1} - \mu_{N,2} - \mu_{N,7} = -4.64... \cdot 10^{-1724}$	$14 - 1.59... \cdot 10^{-6}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 9600$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
9	$-3.14\ldots \cdot 10^{-1187}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3}$ $= +2.83\ldots \cdot 10^{-1186}$	$9 - 2.59\ldots \cdot 10^{-151}$
10	$-9.07\ldots \cdot 10^{-1339}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +9.07\ldots \cdot 10^{-1338}$	$10 + 1.84\ldots \cdot 10^{-125}$
11	$+1.66\ldots \cdot 10^{-1464}$	$\delta_{N,11} - \mu_{N,1}$ $= -1.83\ldots \cdot 10^{-1463}$	$11 + 2.94\ldots \cdot 10^{-103}$
12	$-4.46\ldots \cdot 10^{-1568}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6}$ $= +5.35\ldots \cdot 10^{-1567}$	$12 + 3.63\ldots \cdot 10^{-86}$
13	$+1.35\ldots \cdot 10^{-1654}$	$\delta_{N,13} - \mu_{N,1}$ $= -1.75\ldots \cdot 10^{-1653}$	$13 - 3.19\ldots \cdot 10^{-70}$
14	$+3.31\ldots \cdot 10^{-1725}$	$\delta_{N,14} - \mu_{N,1} - \mu_{N,2} - \mu_{N,7} = -4.64\ldots \cdot 10^{-1724}$	$14 - 1.59\ldots \cdot 10^{-6}$
15	$+3.78\ldots \cdot 10^{-1732}$	$\delta_{N,15} - \mu_{N,1}$ $= +5.29\ldots \cdot 10^{818}$	$-1.39\ldots \cdot 10^{2550}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 9600$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
9	$-3.14\ldots \cdot 10^{-1187}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +2.83\ldots \cdot 10^{-1186}$	$9 - 2.59\ldots \cdot 10^{-151}$
10	$-9.07\ldots \cdot 10^{-1339}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +9.07\ldots \cdot 10^{-1338}$	$10 + 1.84\ldots \cdot 10^{-125}$
11	$+1.66\ldots \cdot 10^{-1464}$	$\delta_{N,11} - \mu_{N,1} = -1.83\ldots \cdot 10^{-1463}$	$11 + 2.94\ldots \cdot 10^{-103}$
12	$-4.46\ldots \cdot 10^{-1568}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +5.35\ldots \cdot 10^{-1567}$	$12 + 3.63\ldots \cdot 10^{-86}$
13	$+1.35\ldots \cdot 10^{-1654}$	$\delta_{N,13} - \mu_{N,1} = -1.75\ldots \cdot 10^{-1653}$	$13 - 3.19\ldots \cdot 10^{-70}$
14	$+3.31\ldots \cdot 10^{-1725}$	$\delta_{N,14} - \mu_{N,1} - \mu_{N,2} - \mu_{N,7} = -4.64\ldots \cdot 10^{-1724}$	$14 - 1.59\ldots \cdot 10^{-6}$
15	$+3.78\ldots \cdot 10^{-1732}$	$\delta_{N,15} - \mu_{N,1} = +5.29\ldots \cdot 10^{818}$	$-1.39\ldots \cdot 10^{2550}$

$$\frac{\mu_{N,3}}{\mu_{N,15}} = 1 + 3.43\ldots \cdot 10^{-968}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 9600$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
9	$-3.14\ldots \cdot 10^{-1187}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +2.83\ldots \cdot 10^{-1186}$	$9 - 2.59\ldots \cdot 10^{-151}$
10	$-9.07\ldots \cdot 10^{-1339}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +9.07\ldots \cdot 10^{-1338}$	$10 + 1.84\ldots \cdot 10^{-125}$
11	$+1.66\ldots \cdot 10^{-1464}$	$\delta_{N,11} - \mu_{N,1} = -1.83\ldots \cdot 10^{-1463}$	$11 + 2.94\ldots \cdot 10^{-103}$
12	$-4.46\ldots \cdot 10^{-1568}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +5.35\ldots \cdot 10^{-1567}$	$12 + 3.63\ldots \cdot 10^{-86}$
13	$+1.35\ldots \cdot 10^{-1654}$	$\delta_{N,13} - \mu_{N,1} = -1.75\ldots \cdot 10^{-1653}$	$13 - 3.19\ldots \cdot 10^{-70}$
14	$+3.31\ldots \cdot 10^{-1725}$	$\delta_{N,14} - \mu_{N,1} - \mu_{N,2} - \mu_{N,7} = -4.64\ldots \cdot 10^{-1724}$	$14 - 1.59\ldots \cdot 10^{-6}$
15	$+3.78\ldots \cdot 10^{-1732}$	$\delta_{N,15} - \mu_{N,1} - \mu_{N,3} = -1.81\ldots \cdot 10^{-149}$	$+4.80\ldots \cdot 10^{1582}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\qquad \mu_{N,1} = \delta_{N,1} = 1$ $\qquad N = 9600$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
9	$-3.14\ldots \cdot 10^{-1187}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +2.83\ldots \cdot 10^{-1186}$	$9 - 2.59\ldots \cdot 10^{-151}$
10	$-9.07\ldots \cdot 10^{-1339}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +9.07\ldots \cdot 10^{-1338}$	$10 + 1.84\ldots \cdot 10^{-125}$
11	$+1.66\ldots \cdot 10^{-1464}$	$\delta_{N,11} - \mu_{N,1} = -1.83\ldots \cdot 10^{-1463}$	$11 + 2.94\ldots \cdot 10^{-103}$
12	$-4.46\ldots \cdot 10^{-1568}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +5.35\ldots \cdot 10^{-1567}$	$12 + 3.63\ldots \cdot 10^{-86}$
13	$+1.35\ldots \cdot 10^{-1654}$	$\delta_{N,13} - \mu_{N,1} = -1.75\ldots \cdot 10^{-1653}$	$13 - 3.19\ldots \cdot 10^{-70}$
14	$+3.31\ldots \cdot 10^{-1725}$	$\delta_{N,14} - \mu_{N,1} - \mu_{N,2} - \mu_{N,7} = -4.64\ldots \cdot 10^{-1724}$	$14 - 1.59\ldots \cdot 10^{-6}$
15	$+3.78\ldots \cdot 10^{-1732}$	$\delta_{N,15} - \mu_{N,1} - \mu_{N,3} = -1.81\ldots \cdot 10^{-149}$	$+4.80\ldots \cdot 10^{1582}$

$$\frac{\mu_{N,5}}{\mu_{N,15}} = 1 - 1.14\ldots \cdot 10^{-1633}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 9600$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
9	$-3.14\ldots \cdot 10^{-1187}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +2.83\ldots \cdot 10^{-1186}$	$9 - 2.59\ldots \cdot 10^{-151}$
10	$-9.07\ldots \cdot 10^{-1339}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = +9.07\ldots \cdot 10^{-1338}$	$10 + 1.84\ldots \cdot 10^{-125}$
11	$+1.66\ldots \cdot 10^{-1464}$	$\delta_{N,11} - \mu_{N,1} = -1.83\ldots \cdot 10^{-1463}$	$11 + 2.94\ldots \cdot 10^{-103}$
12	$-4.46\ldots \cdot 10^{-1568}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +5.35\ldots \cdot 10^{-1567}$	$12 + 3.63\ldots \cdot 10^{-86}$
13	$+1.35\ldots \cdot 10^{-1654}$	$\delta_{N,13} - \mu_{N,1} = -1.75\ldots \cdot 10^{-1653}$	$13 - 3.19\ldots \cdot 10^{-70}$
14	$+3.31\ldots \cdot 10^{-1725}$	$\delta_{N,14} - \mu_{N,1} - \mu_{N,2} - \mu_{N,7} = -4.64\ldots \cdot 10^{-1724}$	$14 - 1.59\ldots \cdot 10^{-6}$
15	$+3.78\ldots \cdot 10^{-1732}$	$\delta_{N,15} - \mu_{N,1} - \mu_{N,3} - \mu_{N,5} = -2.08\ldots \cdot 10^{-1782}$	$+5.52\ldots \cdot 10^{-51}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 12000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
9	$-5.68... \cdot 10^{-1326}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +5.11... \cdot 10^{-1325}$	$9 + 1.47... \cdot 10^{-198}$
10	$+9.29... \cdot 10^{-1525}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = -9.29... \cdot 10^{-1524}$	$10 - 2.44... \cdot 10^{-161}$
11	$+2.27... \cdot 10^{-1686}$	$\delta_{N,11} - \mu_{N,1} = -2.50... \cdot 10^{-1685}$	$11 + 4.37... \cdot 10^{-138}$
12	$-9.06... \cdot 10^{-1825}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +1.08... \cdot 10^{-1823}$	$12 - 4.96... \cdot 10^{-114}$
13	$-3.75... \cdot 10^{-1939}$	$\delta_{N,13} - \mu_{N,1} = +4.88... \cdot 10^{-1938}$	$13 + 1.14... \cdot 10^{-96}$
14	$+3.29... \cdot 10^{-2036}$	$\delta_{N,14} - \mu_{N,1} - \mu_{N,2} - \mu_{N,7} = -4.61... \cdot 10^{-2035}$	$14 - 3.93... \cdot 10^{-81}$
15	$+9.24... \cdot 10^{-2118}$	$\delta_{N,15} - \mu_{N,1} - \mu_{N,3} - \mu_{N,5} = -1.38... \cdot 10^{-2116}$	$15 - 7.70... \cdot 10^{-27}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 12000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
9	$-5.68... \cdot 10^{-1326}$	$\delta_{N,9} - \mu_{N,1} - \mu_{N,3} = +5.11... \cdot 10^{-1325}$	$9 + 1.47... \cdot 10^{-198}$
10	$+9.29... \cdot 10^{-1525}$	$\delta_{N,10} - \mu_{N,1} - \mu_{N,2} - \mu_{N,5} = -9.29... \cdot 10^{-1524}$	$10 - 2.44... \cdot 10^{-161}$
11	$+2.27... \cdot 10^{-1686}$	$\delta_{N,11} - \mu_{N,1} = -2.50... \cdot 10^{-1685}$	$11 + 4.37... \cdot 10^{-138}$
12	$-9.06... \cdot 10^{-1825}$	$\delta_{N,12} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} - \mu_{N,4} - \mu_{N,6} = +1.08... \cdot 10^{-1823}$	$12 - 4.96... \cdot 10^{-114}$
13	$-3.75... \cdot 10^{-1939}$	$\delta_{N,13} - \mu_{N,1} = +4.88... \cdot 10^{-1938}$	$13 + 1.14... \cdot 10^{-96}$
14	$+3.29... \cdot 10^{-2036}$	$\delta_{N,14} - \mu_{N,1} - \mu_{N,2} - \mu_{N,7} = -4.61... \cdot 10^{-2035}$	$14 - 3.93... \cdot 10^{-81}$
15	$+9.24... \cdot 10^{-2118}$	$\delta_{N,15} - \mu_{N,1} - \mu_{N,3} - \mu_{N,5} = -1.38... \cdot 10^{-2116}$	$15 - 7.70... \cdot 10^{-27}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 12000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = +6.21\ldots \cdot 10^{828}$	$-6.21\ldots \cdot 10^{828}$
3	$+3.10\ldots \cdot 10^{828}$	$\delta_{N,3} - \mu_{N,1} = -9.32\ldots \cdot 10^{828}$	$3 + 1.59\ldots \cdot 10^{-276}$
4	$-1.65\ldots \cdot 10^{552}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +6.62\ldots \cdot 10^{552}$	$4 + 4.83\ldots \cdot 10^{-553}$
5	$+1.99\ldots \cdot 10^{-1}$	$\delta_{N,5} - \mu_{N,1} = -1.00\ldots \cdot 10^{0}$	$5 + 2.09\ldots \cdot 10^{-443}$
6	$-8.36\ldots \cdot 10^{-445}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +5.01\ldots \cdot 10^{-444}$	$6 + 1.31\ldots \cdot 10^{-354}$
7	$+1.82\ldots \cdot 10^{-799}$	$\delta_{N,7} - \mu_{N,1} = -1.27\ldots \cdot 10^{-798}$	$7 + 1.09\ldots \cdot 10^{-288}$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 12000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = +6.21\ldots \cdot 10^{828}$	$-6.21\ldots \cdot 10^{828}$
3	$+3.10\ldots \cdot 10^{828}$	$\delta_{N,3} - \mu_{N,1} = -9.32\ldots \cdot 10^{828}$	$3 + 1.59\ldots \cdot 10^{-276}$
4	$-1.65\ldots \cdot 10^{552}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +6.62\ldots \cdot 10^{552}$	$4 + 4.83\ldots \cdot 10^{-553}$
5	$+1.99\ldots \cdot 10^{-1}$	$\delta_{N,5} - \mu_{N,1} = -1.00\ldots \cdot 10^{0}$	$5 + 2.09\ldots \cdot 10^{-443}$
6	$-8.36\ldots \cdot 10^{-445}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +5.01\ldots \cdot 10^{-444}$	$6 + 1.31\ldots \cdot 10^{-354}$
7	$+1.82\ldots \cdot 10^{-799}$	$\delta_{N,7} - \mu_{N,1} = -1.27\ldots \cdot 10^{-798}$	$7 + 1.09\ldots \cdot 10^{-288}$

$$\delta_{12000,2} = 6.218564471518256277409649468 99\ldots \cdot 10^{828}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 12000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = +6.21\ldots \cdot 10^{828}$	$-6.21\ldots \cdot 10^{828}$
3	$+3.10\ldots \cdot 10^{828}$	$\delta_{N,3} - \mu_{N,1} = -9.32\ldots \cdot 10^{828}$	$3 + 1.59\ldots \cdot 10^{-276}$
4	$-1.65\ldots \cdot 10^{552}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +6.62\ldots \cdot 10^{552}$	$4 + 4.83\ldots \cdot 10^{-553}$
5	$+1.99\ldots \cdot 10^{-1}$	$\delta_{N,5} - \mu_{N,1} = -1.00\ldots \cdot 10^{0}$	$5 + 2.09\ldots \cdot 10^{-443}$
6	$-8.36\ldots \cdot 10^{-445}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +5.01\ldots \cdot 10^{-444}$	$6 + 1.31\ldots \cdot 10^{-354}$
7	$+1.82\ldots \cdot 10^{-799}$	$\delta_{N,7} - \mu_{N,1} = -1.27\ldots \cdot 10^{-798}$	$7 + 1.09\ldots \cdot 10^{-288}$

$$\delta_{12000,2} = 6.2185644715182562774096494689 9\ldots \cdot 10^{828}$$

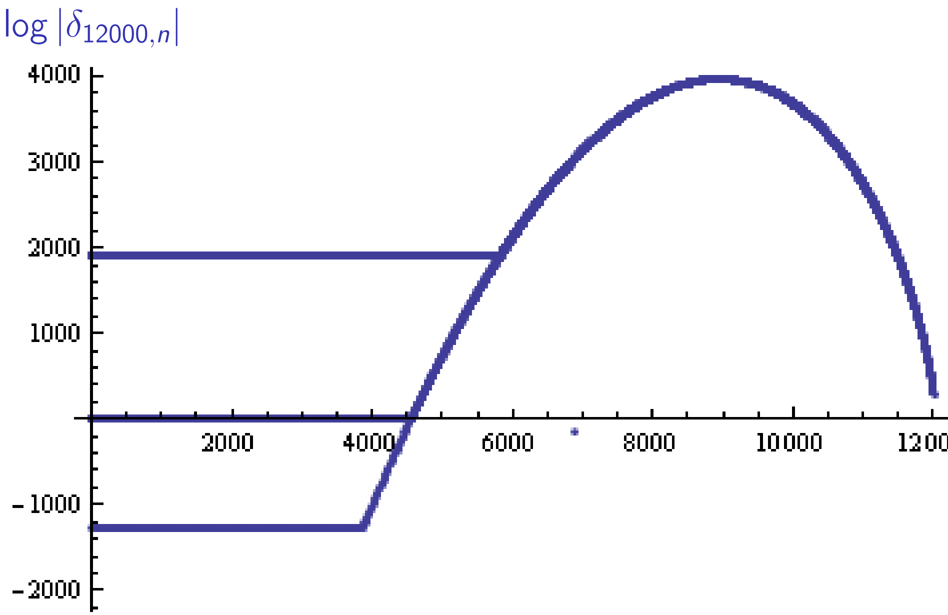
$$\delta_{12000,3} = -9.3278467072773844161144742034 8\ldots \cdot 10^{828}$$

$\nu_{N,n} = \sum_{k=1}^{n-1} \frac{\mu_{N,k}}{k}$ $\mu_{N,1} = \delta_{N,1} = 1$ $N = 12000$

n	$\nu_{N,n}$	$\mu_{N,n}$	$\nu_{N,n}/\mu_{N,n}$
2	1	$\delta_{N,2} - \mu_{N,1} = +6.21\ldots \cdot 10^{828}$	$-6.21\ldots \cdot 10^{828}$
3	$+3.10\ldots \cdot 10^{828}$	$\delta_{N,3} - \mu_{N,1} = -9.32\ldots \cdot 10^{828}$	$3 + 1.59\ldots \cdot 10^{-276}$
4	$-1.65\ldots \cdot 10^{552}$	$\delta_{N,4} - \mu_{N,1} - \mu_{N,2} = +6.62\ldots \cdot 10^{552}$	$4 + 4.83\ldots \cdot 10^{-553}$
5	$+1.99\ldots \cdot 10^{-1}$	$\delta_{N,5} - \mu_{N,1} = -1.00\ldots \cdot 10^{0}$	$5 + 2.09\ldots \cdot 10^{-443}$
6	$-8.36\ldots \cdot 10^{-445}$	$\delta_{N,6} - \mu_{N,1} - \mu_{N,2} - \mu_{N,3} = +5.01\ldots \cdot 10^{-444}$	$6 + 1.31\ldots \cdot 10^{-354}$
7	$+1.82\ldots \cdot 10^{-799}$	$\delta_{N,7} - \mu_{N,1} = -1.27\ldots \cdot 10^{-798}$	$7 + 1.09\ldots \cdot 10^{-288}$

$$\delta_{12000,2} = 6.2185644715182562774096494689 9\ldots \cdot 10^{828}$$
$$\delta_{12000,3} = -9.3278467072773844161144742034 8\ldots \cdot 10^{828}$$
$$\delta_{12000,5} = -2.380857556899868647136714385 96 \cdot 10^{-548}$$

$\log |\delta_{12000,n}|$



$\log|\delta_{12000,n}|$
4000
3000
2000
1000
-1000
-2000
2000
4000
6000
8000
10000
1200