Renormalization-group solutions of nonlinear optics equations

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The problem of light propagation in media exhibiting nonlinear response has been investigated in detail since the early 1960s. Up to now most theoretical results were obtained based on extensive numerical simulation. Analytically one was able to study the problem successfully only for special boundary conditions and the linear dependence of index of refraction on the intensity $n = n_2 I$. For more complicated forms $n(I)$ the problem was investigated only under assumption of fixed (as a rule Gaussian) shape of the beam profile (where the average beam radius, the amplitude and the phase were assumed to be slowly varying functions of the propagation distance). The biased form of the beam profile usually results in a disagreement of analytical predictions with the experiment.

In the present work we obtain approximate analytical solution without imposing any *a priori* restriction on the beam profile in the media. For this goal we used the Lie and the renormalization group theory approach. For different polynomial dependencies of refractive index on intensity we construct analytical solutions for the spatial intensity profile. For the functional forms of the refractive index already studied numerically, we obtain a very good agreement with recent numerical simulations. The derived analytical relations between the self-focusing distance and the index of refraction provide better agreement with the experiment then the methods which are based on an assumption about the beam radial structure.