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Invariant manifolds for physical and chemical kinetics. (English)

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The book "Invariant Manifolds for Physical and Chemical Kinetics" by Alexander N. Gorban and Ilya V. Karlin presents a collection of constructive methods for the study of slow (stable) positively invariant manifolds of finite and infinite dissipative (and partly conservative) dynamic systems that arise in kinetics. Kinetics is mainly described by the Boltzmann equation, a variety of kinetic models such as, e. g., the nonlinear Bhatnagar-Gross-Krook (BGK) model, Fokker-Planck, Enskog, and Vlasov equations, and equations of chemical kinetics which altogether are thoroughly discussed in Chapter 2 of the present book. A major problem that is raised by the Boltzmann equation is the problem of the reduced description that consists in answering the following questions:

1. What are the conditions of the validity of the macroscopic description?
2. Which macroscopic variables are relevant for this description?
3. What are the equations for the macroscopic variables and how they can be derived from the kinetic equations?

The classical methods of the reduced description for the Boltzmann equation are the Hilbert method, the Chapman-Enskog method, and the Grad moment method. The reduction of description suggests the existence of a manifold of slow motions or shortly, a slow manifold in the phase space of the studied dynamic system. It is also assumed that this manifold is positively invariant, that is, if a motion starts on the manifold at $t = t_o$, it retains therein at $t > t_o$. The invariance equation (Chapter 3) that tells that the defect of invariance always vanishes and thus provides the necessary conditions of invariance of a manifold is the key tool, together with the concept of projector field, in the analysis conducted throughout this book. The definition of slowness of a positively invariant manifold is presented in Chapter 4: in fact, a slow invariant manifold is Lyapunov stable fixed point of the film extension of the dynamics. The next Chapter introduces the basic thermodynamic structures, the entropy, the entropic scalar product which is generated by the second differential of the entropy and endows the space of states by the Riemannian structure, the quasi-equilibrium manifold as that of conditional entropy maxima for given values of macroscopic variables, and finally, the thermodynamic projector as the operator that transforms an arbitrary vector field with the given Lyapunov function into a vector field with the same Lyapunov function. This Chapter also provides a number of examples of the Boltzmann equation, covering in particular the Local Maxwellian manifold and the quasi-equilibrium closure hierarchies. The chief aim of the book by A. N. Gorban and I. V. Karlin is to demonstrate how to compute the slow positively invariant manifold by invoking the following three approaches:

(i) Non-perturbative iteration method to solve the invariance equation (Chapter 6). This is the Newton method subject to incomplete linearization. It is rather convenient

to obtain the explicit formulas that is demonstrated by the series of examples including the non-perturbative correction to the Local Maxwellian manifold and the equations of the high-order (post-Navier-Stokes) hydrodynamics;
(ii) Relaxation methods that are based on the stepwise solution of the equation governing the film extension of the original dynamic system (Chapter 9). This method is alternative to the Newton method and more suitable for numerical implementation;
(iii) The method of natural projector that projects not the vector fields but rather finite segments of trajectories (Chapter 11). This method can be considered as the successor of the Ehrenfest model of dynamics with a coarse-graining of the original conservative system to introduce irreversibility and the Hilbert method of solving the Boltzmann equation by constructing the macroscopic equations from the microscopic ones via introducing the parameter that determines the time between coarse-graining (shaking) steps and can be obtained either from the experimental data or phenomenologically. The method of natural projector can be applied to both reversible and irreversible systems. This method is illustrated by the one-particle Liouville equation and the derivation of the fluctuation-dissipation formula.
The last Chapters of the book are devoted to the formulation of the general geometrical framework of nonequilibrium thermodynamics which generalizes the method of natural projector to larger time steps, to the theory of the slow invariant manifolds of weakly open systems, to the methods that allow to estimate the dimension of attractors of an infinite-dimensional system, and to the simple and attractive idea of the post-processing algorithms aimed to control and to enlarge approximate invariant manifolds. As a whole, “Invariant Manifolds for Physical and Chemical Kinetics” is a valuable book to have and to study for everyone who is interested and works in the multi-faceted area of kinetics, either on its mathematical or physical, or chemical sides, with a sufficient background in elementary functional analysis, theory of differential equations, and thermodynamics. The reader may take different tours the authors offer to read their book: the short or long formal roads, or the short and long Boltzmann roads, or the nonequilibrium thermodynamic road, or even the short Grad road. Any road the reader might take would be interesting, useful, and joyful, and if so, the reader may take the advice of Lev D. Landau that if ‘a book is interesting . . . , close the book and try to write it . . . ’ (see p. IX of Preface). I do believe the authors may expect many of such “copies”!

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Keywords : Boltzmann equation; Hilbert method; Chapman-Enskog method; Grad moment method; lattice gas; Fokker-Planck equation; Vlasov equation; reduced description; slowness; entropy; quasiequilibrium; projector field; Newton method; invariance equation; relaxation methods; method of invariant grids; method of natural projector; irreversibility; slow positively invariant manifold; open system; attractors
Classification :
∗82C03 Foundations of time-dependent statistical mechanics
82C05 Classical dynamic and nonequilibrium statistical mechanics (general)
82B03 Foundations of equilibrium statistical mechanics
76P05 Molecular or atomic structure
82B40 Kinetic theory of gases
82C28 Dynamic renormalization group methods
82C31 Stochastic methods in time-dependent statistical mechanics

82C40 Kinetic theory of gases
80A30 Chemical kinetics
80A05 Foundations of classical thermodynamics
60H10 Stochastic ordinary differential equations

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