Model reduction of higher-order KdV equations describing waves on a layer of shallow water

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We study solutions of a generalized non-integrable KdV equation proposed by Fokas [A.S. Fokas, Physica D87, 145 (1995)], aiming to describe unidirectional waves in shallow water to a higher degree of accuracy than the standard KdV equation. This generalized equation includes higher order terms in the small parameters α and β, representing respectively the height and inverse length of the wave compared to the thickness of the water layer. Keeping successively terms of 2^{nd} and 3d order, we reduce the model to 2 and 3-dimensional systems of ODEs and study their invariant manifolds to locate homoclinic and periodic solutions. The solitary waves we find have a smaller height and a larger width than the corresponding KdV soliton, at the same propagation velocity. Extrapolating these results, we conjecture which shape the solitary waves will assume in the limit when terms of arbitrarily high order in α and β are taken into account.